Chapter 2
Graphs

Section 2.1

1. 0

2. $|5-(-3)| = |8| = 8$

3. $\sqrt{3^2+4^2} = \sqrt{25} = 5$

4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

5. $\frac{1}{2}bh$

6. true

7. x-coordinate; y-coordinate

8. quadrants

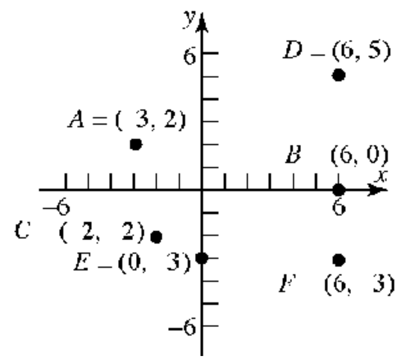
9. midpoint

10. False; the distance between two points is never negative.

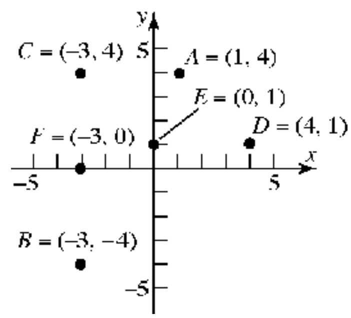
11. False; points that lie in Quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1, 4)$ lies in Quadrant II.

12. True; $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

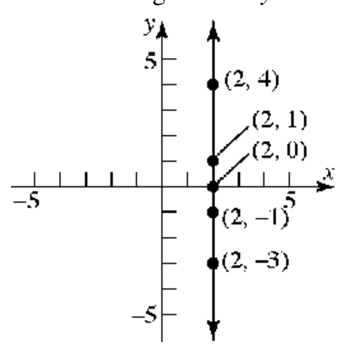
13. (a) Quadrant II
(b) x-axis
(c) Quadrant III
(d) Quadrant I
(e) y-axis
(f) Quadrant IV



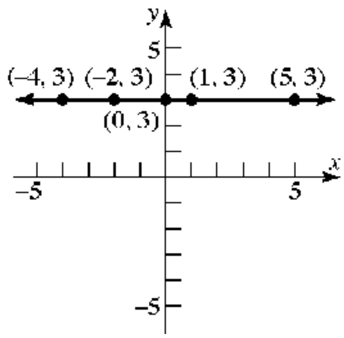
14. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y-axis
(f) x-axis



15. The points will be on a vertical line that is two units to the right of the y-axis.



16. The points will be on a horizontal line that is three units above the *x*-axis.



17. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}$
$=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}$

18. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}$
$=\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}$

19. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}$
$=\sqrt{(-3)^2+1^2}=\sqrt{9+1}=\sqrt{10}$

20. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(2-1)^2}$
$=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$

21. $d(P_1,P_2)=\sqrt{(5-3)^2+\left(4-(-4)\right)^2}$
$=\sqrt{2^2+(8)^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

22. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+\left(4-0\right)^2}$
$=\sqrt{(3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

23. $d(P_1,P_2)=\sqrt{\left(6-(-3)\right)^2+(0-2)^2}$
$=\sqrt{9^2+(-2)^2}=\sqrt{81+4}=\sqrt{85}$

24. $d(P_1,P_2)=\sqrt{\left(4-2\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$

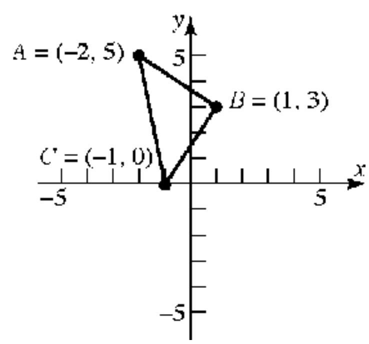
25. $d(P_1,P_2)=\sqrt{(6-4)^2+\left(4-(-3)\right)^2}$
$=\sqrt{2^2+7^2}=\sqrt{4+49}=\sqrt{53}$

26. $d(P_1,P_2)=\sqrt{\left(6-(-4)\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{10^2+5^2}=\sqrt{100+25}$
$=\sqrt{125}=5\sqrt{5}$

27. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}$
$=\sqrt{(-a)^2+(-b)^2}=\sqrt{a^2+b^2}$

28. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}$
$=\sqrt{(-a)^2+(-a)^2}$
$=\sqrt{a^2+a^2}=\sqrt{2a^2}=|\,a\,|\sqrt{2}$

29. $A=(-2,5),\ \ B=(1,3),\ \ C=(-1,0)$
$d(A,B)=\sqrt{\left(1-(-2)\right)^2+(3-5)^2}$
$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$
$d(B,C)=\sqrt{\left(-1-1\right)^2+(0-3)^2}$
$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$
$d(A,C)=\sqrt{\left(-1-(-2)\right)^2+(0-5)^2}$
$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

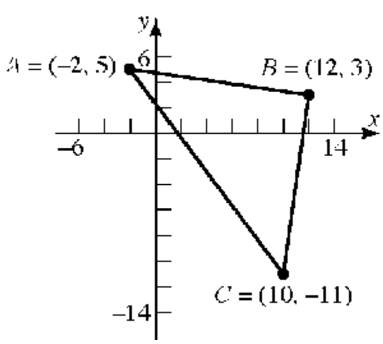
$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2=\left(\sqrt{26}\right)^2$$
$$13+13=26$$
$$26=26$$

The area of a triangle is $A=\frac{1}{2}\cdot bh$. In this problem,

$$A = \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$= \frac{1}{2}\cdot\sqrt{13}\cdot\sqrt{13} = \frac{1}{2}\cdot 13$$
$$= \frac{13}{2} \text{ square units}$$

30. $A = (-2, 5),\ B = (12, 3),\ C = (10, -11)$

$$d(A,B) = \sqrt{(12-(-2))^2 + (3-5)^2}$$
$$= \sqrt{14^2 + (-2)^2}$$
$$= \sqrt{196+4} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(B,C) = \sqrt{(10-12)^2 + (-11-3)^2}$$
$$= \sqrt{(-2)^2 + (-14)^2}$$
$$= \sqrt{4+196} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(A,C) = \sqrt{(10-(-2))^2 + (-11-5)^2}$$
$$= \sqrt{12^2 + (-16)^2}$$
$$= \sqrt{144+256} = \sqrt{400}$$
$$= 20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

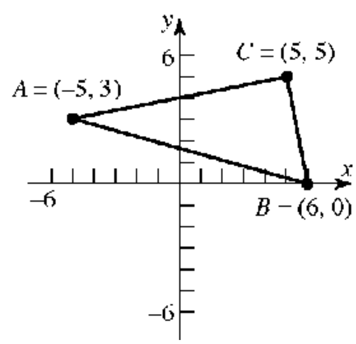
$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$\left(10\sqrt{2}\right)^2 + \left(10\sqrt{2}\right)^2 = (20)^2$$
$$200 + 200 = 400$$
$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2}\cdot[d(A,B)]\cdot[d(B,C)]$$
$$= \frac{1}{2}\cdot 10\sqrt{2}\cdot 10\sqrt{2}$$
$$= \frac{1}{2}\cdot 100\cdot 2$$
$$= 100 \text{ square units}$$

31. $A = (-5,3),\ B = (6,0),\ C = (5,5)$

$$d(A,B) = \sqrt{(6-(-5))^2 + (0-3)^2}$$
$$= \sqrt{11^2 + (-3)^2} = \sqrt{121+9}$$
$$= \sqrt{130}$$
$$d(B,C) = \sqrt{(5-6)^2 + (5-0)^2}$$
$$= \sqrt{(-1)^2 + 5^2} = \sqrt{1+25}$$
$$= \sqrt{26}$$
$$d(A,C) = \sqrt{(5-(-5))^2 + (5-3)^2}$$
$$= \sqrt{10^2 + 2^2} = \sqrt{100+4}$$
$$= \sqrt{104}$$
$$= 2\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

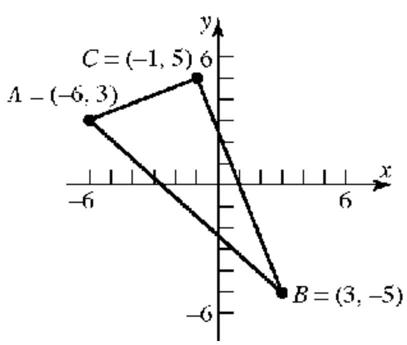
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{104}\right)^2 + \left(\sqrt{26}\right)^2 = \left(\sqrt{130}\right)^2$$
$$104 + 26 = 130$$
$$130 = 130$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot \sqrt{104} \cdot \sqrt{26}$$
$$= \frac{1}{2} \cdot 2\sqrt{26} \cdot \sqrt{26}$$
$$= \frac{1}{2} \cdot 2 \cdot 26$$
$$= 26 \text{ square units}$$

32. $A = (-6, 3), \; B = (3, -5), \; C = (-1, 5)$

$$d(A,B) = \sqrt{(3-(-6))^2 + (-5-3)^2}$$
$$= \sqrt{9^2 + (-8)^2} = \sqrt{81+64}$$
$$= \sqrt{145}$$
$$d(B,C) = \sqrt{(-1-3)^2 + (5-(-5))^2}$$
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16+100}$$
$$= \sqrt{116} = 2\sqrt{29}$$
$$d(A,C) = \sqrt{(-1-(-6))^2 + (5-3)^2}$$
$$= \sqrt{5^2 + 2^2} = \sqrt{25+4}$$
$$= \sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

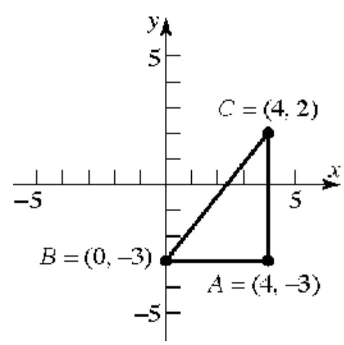
$$[d(A,C)]^2 + [d(B,C)]^2 = [d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2 + \left(2\sqrt{29}\right)^2 = \left(\sqrt{145}\right)^2$$
$$29 + 4 \cdot 29 = 145$$
$$29 + 116 = 145$$
$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2} \cdot [d(A,C)] \cdot [d(B,C)]$$
$$= \frac{1}{2} \cdot \sqrt{29} \cdot 2\sqrt{29}$$
$$= \frac{1}{2} \cdot 2 \cdot 29$$
$$= 29 \text{ square units}$$

33. $A = (4, -3), \; B = (0, -3), \; C = (4, 2)$

$$d(A,B) = \sqrt{(0-4)^2 + (-3-(-3))^2}$$
$$= \sqrt{(-4)^2 + 0^2} = \sqrt{16+0}$$
$$= \sqrt{16}$$
$$= 4$$
$$d(B,C) = \sqrt{(4-0)^2 + (2-(-3))^2}$$
$$= \sqrt{4^2 + 5^2} = \sqrt{16+25}$$
$$= \sqrt{41}$$
$$d(A,C) = \sqrt{(4-4)^2 + (2-(-3))^2}$$
$$= \sqrt{0^2 + 5^2} = \sqrt{0+25}$$
$$= \sqrt{25}$$
$$= 5$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2 + [d(A,C)]^2 = [d(B,C)]^2$$
$$4^2 + 5^2 = \left(\sqrt{41}\right)^2$$
$$16 + 25 = 41$$
$$41 = 41$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

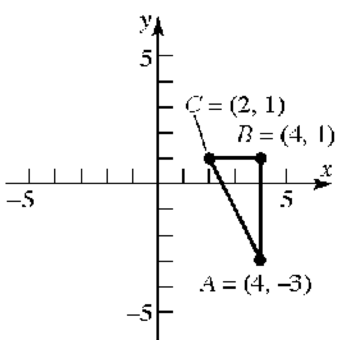
$$A = \frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(A,C)\right] = \frac{1}{2}\cdot 4\cdot 5 = 10 \text{ square units}$$

34. $A=(4,-3),\ B=(4,1),\ C=(2,1)$

$$d(A,B)=\sqrt{(4-4)^2+\left(1-(-3)\right)^2} = \sqrt{0^2+4^2} = \sqrt{0+16} = \sqrt{16} = 4$$

$$d(B,C)=\sqrt{\left(2-4\right)^2+\left(1-1\right)^2} = \sqrt{(-2)^2+0^2} = \sqrt{4+0} = \sqrt{4} = 2$$

$$d(A,C)=\sqrt{(2-4)^2+\left(1-(-3)\right)^2} = \sqrt{(-2)^2+4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\left[d(A,B)\right]^2+\left[d(B,C)\right]^2=\left[d(A,C)\right]^2$$
$$4^2+2^2=\left(2\sqrt{5}\right)^2$$
$$16+4=20$$
$$20=20$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right] = \frac{1}{2}\cdot 4\cdot 2 = 4 \text{ square units}$$

35. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{3+5}{2},\frac{-4+4}{2}\right) = \left(\frac{8}{2},\frac{0}{2}\right) = (4,0)$$

36. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{-2+2}{2},\frac{0+4}{2}\right) = \left(\frac{0}{2},\frac{4}{2}\right) = \left(0,2\right)$$

37. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{-3+6}{2},\frac{2+0}{2}\right) = \left(\frac{3}{2},\frac{2}{2}\right) = \left(\frac{3}{2},1\right)$$

38. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{2+4}{2},\frac{-3+2}{2}\right) = \left(\frac{6}{2},\frac{-1}{2}\right) = \left(3,-\frac{1}{2}\right)$$

39. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4+6}{2}, \frac{-3+1}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{-2}{2}\right)$$
$$= (5, -1)$$

40. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-4+2}{2}, \frac{-3+2}{2}\right)$$
$$= \left(\frac{-2}{2}, \frac{-1}{2}\right)$$
$$= \left(-1, -\frac{1}{2}\right)$$

41. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{a+0}{2}, \frac{b+0}{2}\right)$$
$$= \left(\frac{a}{2}, \frac{b}{2}\right)$$

42. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{a+0}{2}, \frac{a+0}{2}\right)$$
$$= \left(\frac{a}{2}, \frac{a}{2}\right)$$

43. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

44. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$

45. a. If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be 2+3=5. Thus the other leg will be:

$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = 12$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

b. Consider points of the form $(3, y)$ that are a distance of 13 units from the point $(-2,-1)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-(-2))^2 + (-1-y)^2}$$
$$= \sqrt{(5)^2 + (-1-y)^2}$$
$$= \sqrt{25+1+2y+y^2}$$
$$= \sqrt{y^2+2y+26}$$
$$13 = \sqrt{y^2+2y+26}$$
$$13^2 = \left(\sqrt{y^2+2y+26}\right)^2$$
$$169 = y^2+2y+26$$
$$0 = y^2+2y-143$$
$$0 = (y-11)(y+13)$$
$$y-11=0 \quad \text{or} \quad y+13=0$$
$$y=11 \qquad y=-13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

46. a. If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be 2+6=8. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$
$$64 + b^2 = 289$$
$$b^2 = 225$$
$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14,-6)$ and $(16,-6)$.

b. Consider points of the form $(x,-6)$ that are a distance of 17 units from the point $(1,2)$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(1-x)^2+(2-(-6))^2}$$
$$=\sqrt{x^2-2x+1+(8)^2}$$
$$=\sqrt{x^2-2x+1+64}$$
$$=\sqrt{x^2-2x+65}$$
$$17=\sqrt{x^2-2x+65}$$
$$17^2=\left(\sqrt{x^2-2x+65}\right)^2$$
$$289=x^2-2x+65$$
$$0=x^2-2x-224$$
$$0=(x+14)(x-16)$$
$$x+14=0 \quad \text{or} \quad x-16=0$$
$$x=-14 \qquad x=16$$

Thus, the points $(-14,-6)$ and $(16,-6)$ are a distance of 13 units from the point $(1,2)$.

47. Points on the *x*-axis have a *y*-coordinate of 0. Thus, we consider points of the form $(x,0)$ that are a distance of 6 units from the point $(4,-3)$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(4-x)^2+(-3-0)^2}$$
$$=\sqrt{16-8x+x^2+(-3)^2}$$
$$=\sqrt{16-8x+x^2+9}$$
$$=\sqrt{x^2-8x+25}$$
$$6=\sqrt{x^2-8x+25}$$
$$6^2=\left(\sqrt{x^2-8x+25}\right)^2$$
$$36=x^2-8x+25$$
$$0=x^2-8x-11$$
$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-11)}}{2(1)}$$
$$=\frac{8\pm\sqrt{64+44}}{2}=\frac{8\pm\sqrt{108}}{2}$$
$$=\frac{8\pm6\sqrt{3}}{2}=4\pm3\sqrt{3}$$
$$x=4+3\sqrt{3} \quad \text{or} \quad x=4-3\sqrt{3}$$

Thus, the points $\left(4+3\sqrt{3},0\right)$ and $\left(4-3\sqrt{3},0\right)$ are on the *x*-axis and a distance of 6 units from the point $(4,-3)$.

48. Points on the *y*-axis have an *x*-coordinate of 0. Thus, we consider points of the form $(0,y)$ that are a distance of 6 units from the point $(4,-3)$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(4-0)^2+(-3-y)^2}$$
$$=\sqrt{4^2+9+6y+y^2}$$
$$=\sqrt{16+9+6y+y^2}$$
$$=\sqrt{y^2+6y+25}$$
$$6=\sqrt{x^2+6x+25}$$
$$6^2=\left(\sqrt{x^2+6x+25}\right)^2$$
$$36=x^2+6x+25$$
$$0=x^2+6x-11$$
$$x=\frac{(-6)\pm\sqrt{(6)^2-4(1)(-11)}}{2(1)}$$
$$=\frac{-6\pm\sqrt{36+44}}{2}=\frac{-6\pm\sqrt{80}}{2}$$
$$=\frac{-6\pm4\sqrt{5}}{2}=-3\pm2\sqrt{5}$$
$$x=-6+2\sqrt{5} \quad \text{or} \quad x=-6-2\sqrt{5}$$

Thus, the points $\left(0,-6+2\sqrt{5}\right)$ and $\left(0,-6-2\sqrt{5}\right)$

are on the y-axis and a distance of 6 units from the point $(4,-3)$.

49. $M=(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

$P_1=(x_1,y_1)=(-3,6)$ and $(x,y)=(-1,4)$, so

$$x=\frac{x_1+x_2}{2} \quad \text{and} \quad y=\frac{y_1+y_2}{2}$$
$$-1=\frac{-3+x_2}{2} \qquad 4=\frac{6+y_2}{2}$$
$$-2=-3+x_2 \qquad 8=6+y_2$$
$$1=x_2 \qquad 2=y_2$$

Thus, $P_2=(1,2)$.

50. $M=(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

$P_2=(x_2,y_2)=(7,-2)$ and $(x,y)=(5,-4)$, so

$$x=\frac{x_1+x_2}{2} \quad \text{and} \quad y=\frac{y_1+y_2}{2}$$
$$5=\frac{x_1+7}{2} \qquad -4=\frac{y_1+(-2)}{2}$$
$$10=x_1+7 \qquad -8=y_1+(-2)$$
$$3=x_1 \qquad -6=y_1$$

Thus, $P_1=(3,-6)$.

51. The midpoint of AB is: $D=\left(\frac{0+6}{2},\frac{0+0}{2}\right)=(3,0)$

The midpoint of AC is: $E=\left(\frac{0+4}{2},\frac{0+4}{2}\right)=(2,2)$

The midpoint of BC is: $F=\left(\frac{6+4}{2},\frac{0+4}{2}\right)=(5,2)$

$$d(C,D)=\sqrt{(0-4)^2+(3-4)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$

$$d(B,E)=\sqrt{(2-6)^2+(2-0)^2}=\sqrt{(-4)^2+2^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

$$d(A,F)=\sqrt{(2-0)^2+(5-0)^2}=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$$

52. Let $P_1=(0,0)$, $P_2=(0,4)$, $P=(x,y)$

$$d(P_1,P_2)=\sqrt{(0-0)^2+(4-0)^2}=\sqrt{16}=4$$
$$d(P_1,P)=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}=4$$
$$\to\ x^2+y^2=16$$
$$d(P_2,P)=\sqrt{(x-0)^2+(y-4)^2}=\sqrt{x^2+(y-4)^2}=4$$
$$\to\ x^2+(y-4)^2=16$$

Therefore,

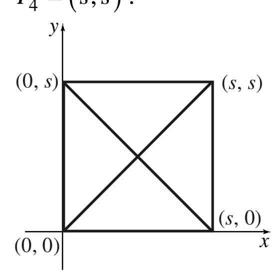
$$y^2=(y-4)^2$$
$$y^2=y^2-8y+16$$
$$8y=16$$
$$y=2$$

which gives

$$x^2+2^2=16$$
$$x^2=12$$
$$x=\pm 2\sqrt{3}$$

Two triangles are possible. The third vertex is $(-2\sqrt{3},2)$ or $(2\sqrt{3},2)$.

53. Let $P_1=(0,0)$, $P_2=(0,s)$, $P_3=(s,0)$, and $P_4=(s,s)$.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.

$$M_{1,4}=\left(\frac{0+s}{2},\frac{0+s}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$

$$M_{2,3}=\left(\frac{0+s}{2},\frac{s+0}{2}\right)=\left(\frac{s}{2},\frac{s}{2}\right)$$

The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

54. Let $P_1=(0,0)$, $P_2=(a,0)$, and $P_3=\left(\frac{a}{2},\frac{\sqrt{3}\,a}{2}\right)$. To show that these vertices form an equilateral triangle, we need to show that the distance between any pair of points is the same constant value.

$$d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(a-0)^2+(0-0)^2}=\sqrt{a^2}=|a|$$

$$d(P_2,P_3)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(\frac{a}{2}-a\right)^2+\left(\frac{\sqrt{3}\,a}{2}-0\right)^2}$$
$$=\sqrt{\frac{a^2}{4}+\frac{3a^2}{4}}=\sqrt{\frac{4a^2}{4}}=\sqrt{a^2}=|a|$$

$$d(P_1,P_3)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{\sqrt{3}\,a}{2}-0\right)^2}$$
$$=\sqrt{\frac{a^2}{4}+\frac{3a^2}{4}}=\sqrt{\frac{4a^2}{4}}=\sqrt{a^2}=|a|$$

Since all three distances have the same constant value, the triangle is an equilateral triangle.

Now find the midpoints:

$$P_4=M_{P_1P_2}=\left(\frac{0+a}{2},\frac{0+0}{2}\right)=\left(\frac{a}{2},0\right)$$

$$P_5=M_{P_2P_3}=\left(\frac{a+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}\,a}{2}}{2}\right)=\left(\frac{3a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

$$P_6=M_{P_1P_3}=\left(\frac{0+\frac{a}{2}}{2},\frac{0+\frac{\sqrt{3}a}{2}}{2}\right)=\left(\frac{a}{4},\frac{\sqrt{3}\,a}{4}\right)$$

$$d(P_4,P_5)=\sqrt{\left(\frac{3a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

$$d(P_4,P_6)=\sqrt{\left(\frac{a}{4}-\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-0\right)^2}$$
$$=\sqrt{\left(-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\frac{a^2}{16}+\frac{3a^2}{16}}=\frac{|a|}{2}$$

$$d(P_5,P_6)=\sqrt{\left(\frac{3a}{4}-\frac{a}{4}\right)^2+\left(\frac{\sqrt{3}\,a}{4}-\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$=\sqrt{\left(\frac{a}{2}\right)^2+0^2}$$
$$=\sqrt{\frac{a^2}{4}}=\frac{|a|}{2}$$

Since the sides are the same length, the triangle is equilateral.

55. $d(P_1,P_2)=\sqrt{(-4-2)^2+(1-1)^2}$
$=\sqrt{(-6)^2+0^2}$
$=\sqrt{36}$
$=6$

$d(P_2,P_3)=\sqrt{(-4-(-4))^2+(-3-1)^2}$
$=\sqrt{0^2+(-4)^2}$
$=\sqrt{16}$
$=4$

$d(P_1,P_3)=\sqrt{(-4-2)^2+(-3-1)^2}$
$=\sqrt{(-6)^2+(-4)^2}$
$=\sqrt{36+16}$
$=\sqrt{52}$
$=2\sqrt{13}$

Since $[d(P_1,P_2)]^2+[d(P_2,P_3)]^2=[d(P_1,P_3)]^2$, the triangle is a right triangle.

56. $d(P_1,P_2)=\sqrt{(6-(-1))^2+(2-4)^2}$
$=\sqrt{7^2+(-2)^2}$
$=\sqrt{49+4}$
$=\sqrt{53}$

$d(P_2,P_3)=\sqrt{(4-6)^2+(-5-2)^2}$
$=\sqrt{(-2)^2+(-7)^2}$
$=\sqrt{4+49}$
$=\sqrt{53}$

$d(P_1,P_3)=\sqrt{(4-(-1))^2+(-5-4)^2}$
$=\sqrt{5^2+(-9)^2}$
$=\sqrt{25+81}$
$=\sqrt{106}$

Since $[d(P_1,P_2)]^2+[d(P_2,P_3)]^2=[d(P_1,P_3)]^2$, the triangle is a right triangle.

Since $d(P_1,P_2)=d(P_2,P_3)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

57. $d(P_1,P_2)=\sqrt{(0-(-2))^2+(7-(-1))^2}$
$=\sqrt{2^2+8^2}=\sqrt{4+64}=\sqrt{68}$
$=2\sqrt{17}$

$d(P_2,P_3)=\sqrt{(3-0)^2+(2-7)^2}$
$=\sqrt{3^2+(-5)^2}=\sqrt{9+25}$
$=\sqrt{34}$

$d(P_1,P_3)=\sqrt{(3-(-2))^2+(2-(-1))^2}$
$=\sqrt{5^2+3^2}=\sqrt{25+9}$
$=\sqrt{34}$

Since $d(P_2,P_3)=d(P_1,P_3)$, the triangle is isosceles.

Since $[d(P_1,P_3)]^2+[d(P_2,P_3)]^2=[d(P_1,P_2)]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

58. $d(P_1,P_2)=\sqrt{(-4-7)^2+(0-2)^2}$
$=\sqrt{(-11)^2+(-2)^2}$
$=\sqrt{121+4}=\sqrt{125}$
$=5\sqrt{5}$

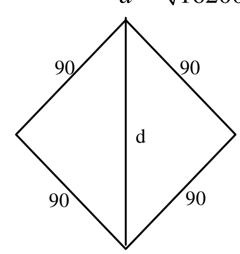
$d(P_2,P_3)=\sqrt{(4-(-4))^2+(6-0)^2}$
$=\sqrt{8^2+6^2}=\sqrt{64+36}$
$=\sqrt{100}$
$=10$

$d(P_1,P_3)=\sqrt{(4-7)^2+(6-2)^2}$
$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Since $[d(P_1,P_3)]^2+[d(P_2,P_3)]^2=[d(P_1,P_2)]^2$, the triangle is a right triangle.

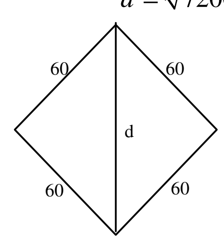
59. Using the Pythagorean Theorem:

$90^2+90^2=d^2$
$8100+8100=d^2$
$16200=d^2$
$d=\sqrt{16200}=90\sqrt{2}\approx 127.28$ feet

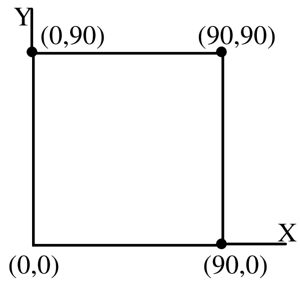


60. Using the Pythagorean Theorem:

$60^2+60^2=d^2$
$3600+3600=d^2 \rightarrow 7200=d^2$
$d=\sqrt{7200}=60\sqrt{2}\approx 84.85$ feet



61. a. First: (90, 0), Second: (90, 90), Third: (0, 90)



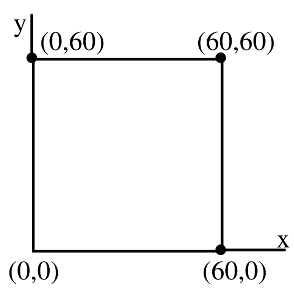
b. Using the distance formula:

$$d = \sqrt{(310-90)^2 + (15-90)^2}$$
$$= \sqrt{220^2 + (-75)^2} = \sqrt{54025}$$
$$= 5\sqrt{2161} \approx 232.43 \text{ feet}$$

c. Using the distance formula:

$$d = \sqrt{(300-0)^2 + (300-90)^2}$$
$$= \sqrt{300^2 + 210^2} = \sqrt{134100}$$
$$= 30\sqrt{149} \approx 366.20 \text{ feet}$$

62. a. First: (60, 0), Second: (60, 60)
Third: (0, 60)



b. Using the distance formula:

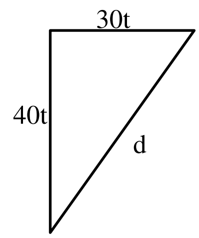
$$d = \sqrt{(180-60)^2 + (20-60)^2}$$
$$= \sqrt{120^2 + (-40)^2} = \sqrt{16000}$$
$$= 40\sqrt{10} \approx 126.49 \text{ feet}$$

c. Using the distance formula:

$$d = \sqrt{(220-0)^2 + (220-60)^2}$$
$$= \sqrt{220^2 + 160^2} = \sqrt{74000}$$
$$= 20\sqrt{185} \approx 272.03 \text{ feet}$$

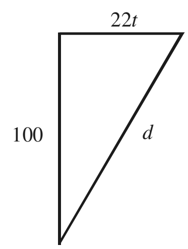
63. The Neon heading east moves a distance $30t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$d = \sqrt{(30t)^2 + (40t)^2}$$
$$= \sqrt{900t^2 + 1600t^2}$$
$$= \sqrt{2500t^2}$$
$$= 50t \text{ miles}$$



64. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$d = \sqrt{100^2 + (22t)^2}$$
$$= \sqrt{10000 + 484t^2} \text{ feet}$$



65. a. The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2}\right)$$
$$= \left(\frac{5.3}{2}, \frac{3.2}{2}\right)$$
$$= (2.65, 1.6)$$

b. Using the distance formula:

$$d = \sqrt{(2.65-1.4)^2 + (1.6-1.3)^2}$$
$$= \sqrt{(1.25)^2 + (0.3)^2}$$
$$= \sqrt{1.5625 + 0.09}$$
$$= \sqrt{1.6525}$$
$$\approx 1.285 \text{ units}$$

66. Let $P_1 = (2002, 204)$ and $P_2 = (2008, 375)$. The midpoint is:

$$\begin{aligned}(x,\ y) &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\\ &= \left(\frac{2002+2008}{2}, \frac{204+375}{2}\right)\\ &= \left(\frac{4010}{2}, \frac{579}{2}\right)\\ &= (2005,\ 289.5)\end{aligned}$$

The estimate for 2005 is \$289.5 billion. The estimate net sales of Wal-Mart Stores, Inc. in 2005 is \$7.5 billion off from the reported value of \$282 billion.

67. For 1998 we have the ordered pair $(1998, 16530)$ and for 2008 we have the ordered pair $(2008, 21834)$. The midpoint is

$$\begin{aligned}(\text{year}, \$) &= \left(\frac{1998+2008}{2}, \frac{16530+21834}{2}\right)\\ &= \left(\frac{4006}{2}, \frac{38364}{2}\right)\\ &= (2003, 19182)\end{aligned}$$

Using the midpoint, we estimate the poverty level in 2003 to be \$19,182. This is slightly higher than the actual value.

68. Answers will vary.

Section 2.2

1. $2(x+3)-1=-7$

$$\begin{aligned}2(x+3)&=-6\\ x+3&=-3\\ x&=-6\end{aligned}$$

The solution set is $\{-6\}$.

2. $x^2-9=0$

$$\begin{aligned}x^2&=9\\ x&=\pm\sqrt{9}\\ x&=\pm 3\end{aligned}$$

The solution set is $\{-3, 3\}$.

3. intercepts

4. $y=0$

5. y-axis

6. 4

7. $(-3, 4)$

8. True

9. False; the y-coordinate of a point at which the graph crosses or touches the x-axis is always 0. The x-coordinate of such a point is an x-intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).
For example: $x^2+y^2=1$

11. $y=x^4-\sqrt{x}$

$0=0^4-\sqrt{0}$ $1=1^4-\sqrt{1}$ $0=(-1)^4-\sqrt{-1}$

$0=0$ $1\neq 0$ $0\neq 1-\sqrt{-1}$

The point (0, 0) is on the graph of the equation.

12. $y=x^3-2\sqrt{x}$

$0=0^3-2\sqrt{0}$ $1=1^3-2\sqrt{1}$ $-1=1^3-2\sqrt{1}$

$0=0$ $1\neq -1$ $-1=-1$

The points (0, 0) and (1, –1) are on the graph of the equation.

13. $y^2=x^2+9$

$3^2=0^2+9$ $0^2=3^2+9$ $0^2=(-3)^2+9$

$9=9$ $0\neq 18$ $0\neq 18$

The point (0, 3) is on the graph of the equation.

14. $y^3=x+1$

$2^3=1+1$ $1^3=0+1$ $0^3=-1+1$

$8\neq 2$ $1=1$ $0=0$

The points (0, 1) and (–1, 0) are on the graph of the equation.

15. $x^2+y^2=4$

$0^2+2^2=4$ $(-2)^2+2^2=4$ $\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2=4$

$4=4$ $8\neq 4$ $4=4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

16. $x^2+4y^2=4$

$0^2+4\cdot 1^2=4$ $\quad$ $2^2+4\cdot 0^2=4$ $\quad$ $2^2+4\left(\frac{1}{2}\right)^2=4$

$4=4$ $\quad$ $4=4$ $\quad$ $5\neq 4$

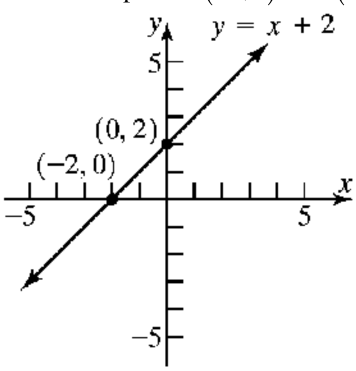
The points (0, 1) and (2, 0) are on the graph of the equation.

17. $y=x+2$

x-intercept: $0=x+2$, $-2=x$

y-intercept: $y=0+2$, $y=2$

The intercepts are $(-2,0)$ and $(0,2)$.

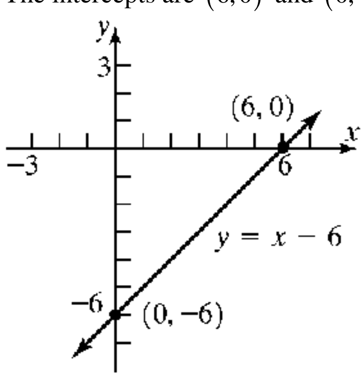


18. $y=x-6$

x-intercept: $0=x-6$, $6=x$

y-intercept: $y=0-6$, $y=-6$

The intercepts are $(6,0)$ and $(0,-6)$.

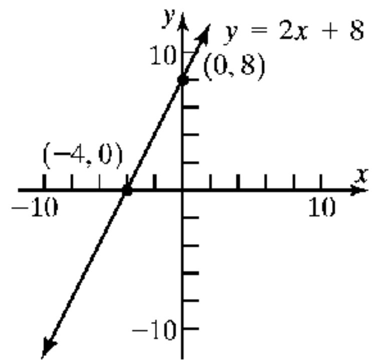


19. $y=2x+8$

x-intercept: $0=2x+8$, $2x=-8$, $x=-4$

y-intercept: $y=2(0)+8$, $y=8$

The intercepts are $(-4,0)$ and $(0,8)$.

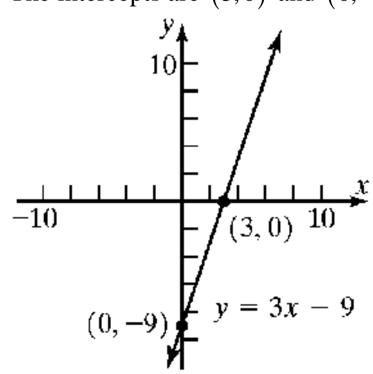


20. $y=3x-9$

x-intercept: $0=3x-9$, $3x=9$, $x=3$

y-intercept: $y=3(0)-9$, $y=-9$

The intercepts are $(3,0)$ and $(0,-9)$.

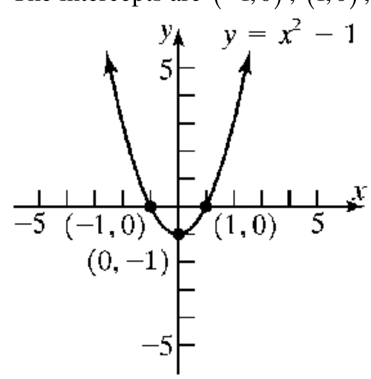


21. $y=x^2-1$

x-intercepts: $0=x^2-1$, $x^2=1$, $x=\pm 1$

y-intercept: $y=0^2-1$, $y=-1$

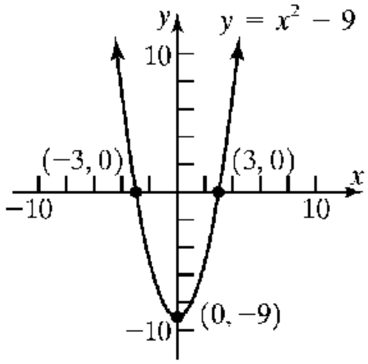
The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.



22. $y = x^2 - 9$

x-intercepts:	y-intercept:
$0 = x^2 - 9$	$y = 0^2 - 9$
$x^2 = 9$	$y = -9$
$x = \pm 3$	

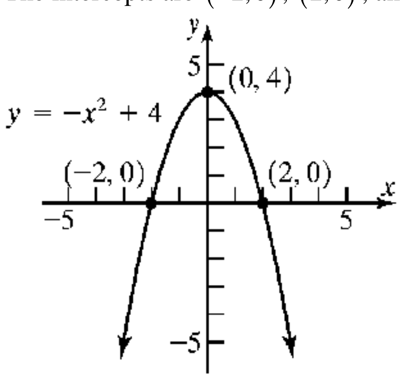
The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.



23. $y = -x^2 + 4$

x-intercepts:	y-intercepts:
$0 = -x^2 + 4$	$y = -(0)^2 + 4$
$x^2 = 4$	$y = 4$
$x = \pm 2$	

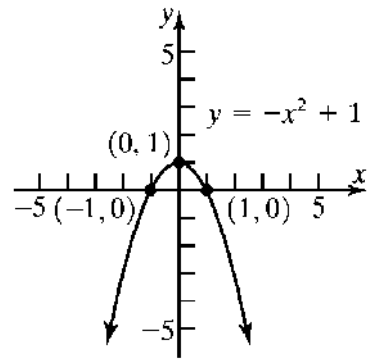
The intercepts are $(-2,0)$, $(2,0)$, and $(0,4)$.



24. $y = -x^2 + 1$

x-intercepts:	y-intercept:
$0 = -x^2 + 1$	$y = -(0)^2 + 1$
$x^2 = 1$	$y = 1$
$x = \pm 1$	

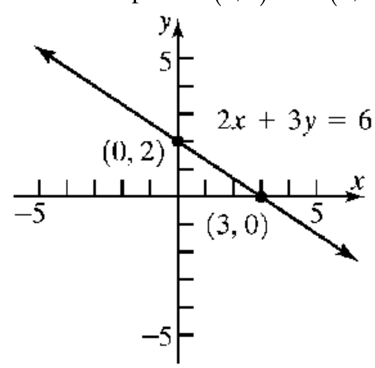
The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.



25. $2x + 3y = 6$

x-intercepts:	y-intercept:
$2x + 3(0) = 6$	$2(0) + 3y = 6$
$2x = 6$	$3y = 6$
$x = 3$	$y = 2$

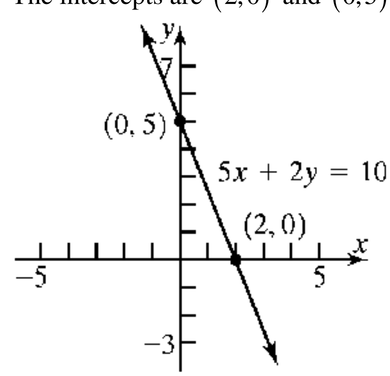
The intercepts are $(3,0)$ and $(0,2)$.



26. $5x + 2y = 10$

x-intercepts:	y-intercept:
$5x + 2(0) = 10$	$5(0) + 2y = 10$
$5x = 10$	$2y = 10$
$x = 2$	$y = 5$

The intercepts are $(2,0)$ and $(0,5)$.



27. $9x^2 + 4y = 36$

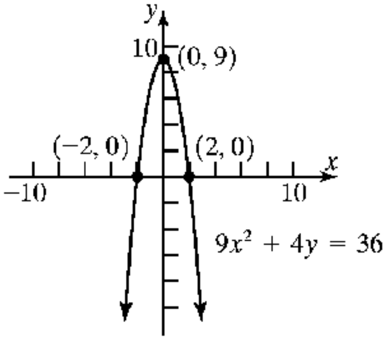
x-intercepts:

$$9x^2 + 4(0) = 36$$
$$9x^2 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

y-intercept:

$$9(0)^2 + 4y = 36$$
$$4y = 36$$
$$y = 9$$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.



28. $4x^2 + y = 4$

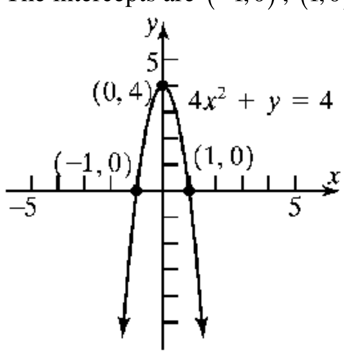
x-intercepts:

$$4x^2 + 0 = 4$$
$$4x^2 = 4$$
$$x^2 = 1$$
$$x = \pm 1$$

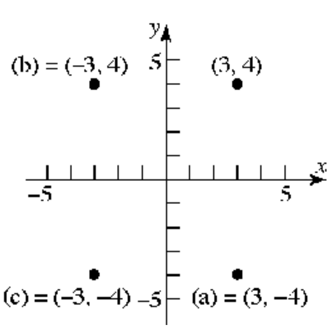
y-intercept:

$$4(0)^2 + y = 4$$
$$y = 4$$

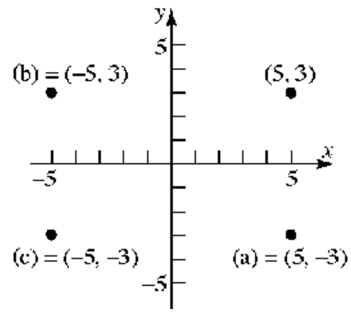
The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



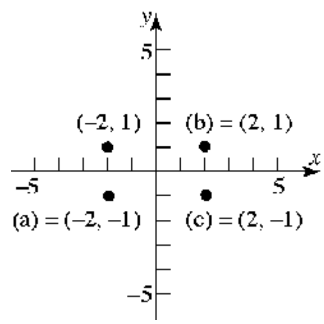
29.



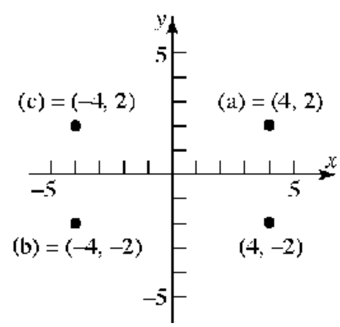
30.



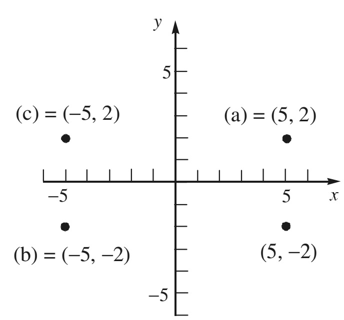
31.



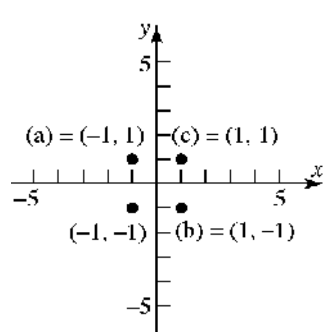
32.



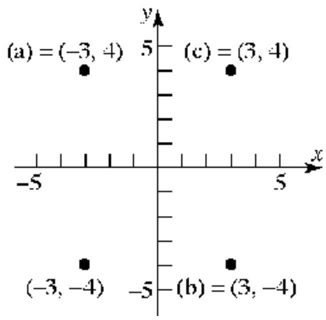
33.



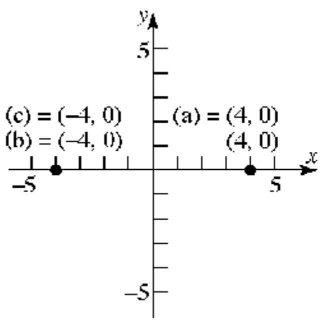
34.



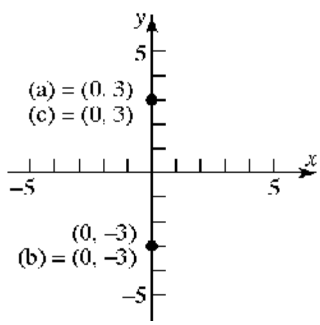
35.



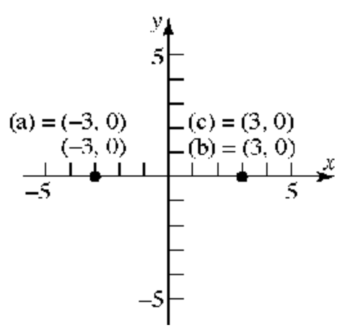
36.



37.



38.



39. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

40. **a.** Intercepts: $(0,1)$

b. Not symmetric to the x-axis, the y-axis, nor the origin

41. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the y-axis.

42. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the y-axis.

43. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the x-axis.

44. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the x-axis, y-axis, and the origin.

45. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

46. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

47. **a.** x-intercept: $[-2,1]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

48. **a.** x-intercept: $[-1,2]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

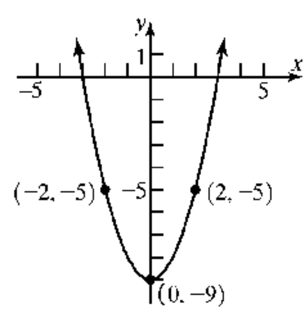
49. **a.** Intercepts: none

b. Symmetric with respect to the origin.

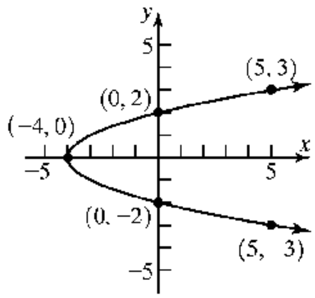
50. **a.** Intercepts: none

b. Symmetric with respect to the x-axis.

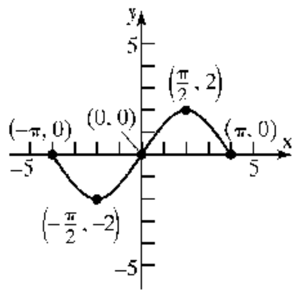
51.



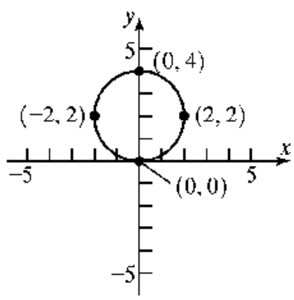
52.



53.



54.



55. $y^2 = x + 4$

x-intercepts: $0^2 = x + 4$, $-4 = x$

y-intercepts: $y^2 = 0 + 4$, $y^2 = 4$, $y = \pm 2$

The intercepts are $(-4,0)$, $(0,-2)$ and $(0,2)$.

Test *x*-axis symmetry: Let $y = -y$

$(-y)^2 = x + 4$

$y^2 = x + 4$ same

Test *y*-axis symmetry: Let $x = -x$

$y^2 = -x + 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 4$

$y^2 = -x + 4$ different

Therefore, the graph will have *x*-axis symmetry.

56. $y^2 = x + 9$

x-intercepts: $(0)^2 = -x + 9$, $0 = -x + 9$, $x = 9$

y-intercepts: $y^2 = 0 + 9$, $y^2 = 9$, $y = \pm 3$

The intercepts are $(-9,0)$, $(0,-3)$ and $(0,3)$.

Test *x*-axis symmetry: Let $y = -y$

$(-y)^2 = x + 9$

$y^2 = x + 9$ same

Test *y*-axis symmetry: Let $x = -x$

$y^2 = -x + 9$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.

$(-y)^2 = -x + 9$

$y^2 = -x + 9$ different

Therefore, the graph will have *x*-axis symmetry.

57. $y = \sqrt[3]{x}$

x-intercepts: $0 = \sqrt[3]{x}$, $0 = x$

y-intercepts: $y = \sqrt[3]{0} = 0$

The only intercept is $(0,0)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = \sqrt[3]{x}$ different

Test *y*-axis symmetry: Let $x = -x$

$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$

$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

58. $y = \sqrt[5]{x}$

x-intercepts: $0 = \sqrt[3]{x}$, $0 = x$

y-intercepts: $y = \sqrt[5]{0} = 0$

The only intercept is $(0,0)$.

Test *x*-axis symmetry: Let $y = -y$

$-y = \sqrt[5]{x}$ different

Test *y*-axis symmetry: Let $x = -x$

$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x=-x$ and $y=-y$

$-y=\sqrt[5]{-x}=-\sqrt[5]{x}$

$y=\sqrt[5]{x}$ same

Therefore, the graph will have origin symmetry.

59. $x^2+y-9=0$

x-intercepts: $x^2-9=0$, $x^2=9$, $x=\pm3$

y-intercepts: $0^2+y-9=0$, $y=9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y=-y$

$x^2-y-9=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2+y-9=0$

$x^2+y-9=0$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$(-x)^2-y-9=0$

$x^2-y-9=0$ different

Therefore, the graph will have y-axis symmetry.

60. $x^2-y-4=0$

x-intercepts: $x^2-0-4=0$, $x^2=4$, $x=\pm2$

y-intercept: $0^2-y-4=0$, $-y=4$, $y=-4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y=-y$

$x^2-(-y)-4=0$

$x^2+y-4=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2-y-4=0$

$x^2-y-4=0$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$(-x)^2-(-y)-4=0$

$x^2+y-4=0$ different

Therefore, the graph will have y-axis symmetry.

61. $9x^2+4y^2=36$

x-intercepts: $9x^2+4(0)^2=36$, $9x^2=36$, $x^2=4$, $x=\pm2$

y-intercepts: $9(0)^2+4y^2=36$, $4y^2=36$, $y^2=9$, $y=\pm3$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y=-y$

$9x^2+4(-y)^2=36$

$9x^2+4y^2=36$ same

Test y-axis symmetry: Let $x=-x$

$9(-x)^2+4y^2=36$

$9x^2+4y^2=36$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$9(-x)^2+4(-y)^2=36$

$9x^2+4y^2=36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

62. $4x^2+y^2=4$

x-intercepts: $4x^2+0^2=4$, $4x^2=4$, $x^2=1$, $x=\pm1$

y-intercepts: $4(0)^2+y^2=4$, $y^2=4$, $y=\pm2$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y=-y$

$4x^2+(-y)^2=4$

$4x^2+y^2=4$ same

Test y-axis symmetry: Let $x=-x$

$4(-x)^2+y^2=4$

$4x^2+y^2=4$ same

Test origin symmetry: Let $x=-x$ and $y=-y$

$4(-x)^2+(-y)^2=4$

$4x^2+y^2=4$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

63. $y=x^3-27$

x-intercepts: y-intercepts:

$0 = x^3 - 27 \qquad y = 0^3 - 27$

$x^3 = 27 \qquad y = -27$

$x = 3$

The intercepts are $(3,0)$ and $(0,-27)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - 27$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - 27$

$y = -x^3 - 27$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^3 - 27$

$y = x^3 + 27$ different

Therefore, the graph has none of the indicated symmetries.

64. $y = x^4 - 1$

x-intercepts: $\qquad$ y-intercepts:

$0 = x^4 - 1 \qquad y = 0^4 - 1$

$x^4 = 1 \qquad y = -1$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 1$

$y = x^4 - 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 1$

$-y = x^4 - 1$ different

Therefore, the graph will have y-axis symmetry.

65. $y = x^2 - 3x - 4$

x-intercepts: $\qquad$ y-intercepts:

$0 = x^2 - 3x - 4 \qquad y = 0^2 - 3(0) - 4$

$0 = (x-4)(x+1) \qquad y = -4$

$x = 4$ or $x = -1$

The intercepts are $(4,0)$, $(-1,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 - 3x - 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 - 3(-x) - 4$

$y = x^2 + 3x - 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 - 3(-x) - 4$

$-y = x^2 + 3x - 4$ different

Therefore, the graph has none of the indicated symmetries.

66. $y = x^2 + 4$

x-intercepts: $\qquad$ y-intercepts:

$0 = x^2 + 4 \qquad y = 0^2 + 4$

$x^2 = -4 \qquad y = 4$

no real solution

The only intercept is $(0,4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 + 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 + 4$

$y = x^2 + 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 + 4$

$-y = x^2 + 4$ different

Therefore, the graph will have y-axis symmetry.

67. $y = \dfrac{3x}{x^2 + 9}$

x-intercepts: $\qquad$ y-intercepts:

$0 = \dfrac{3x}{x^2 + 9} \qquad y = \dfrac{3(0)}{0^2 + 9} = \dfrac{0}{9} = 0$

$3x = 0$

$x = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{3x}{x^2 + 9}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{3(-x)}{(-x)^2 + 9}$

$y = -\dfrac{3x}{x^2 + 9}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{3(-x)}{(-x)^2 + 9}$$

$$-y = -\frac{3x}{x^2 + 9}$$

$$y = \frac{3x}{x^2 + 9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

68. $y = \dfrac{x^2 - 4}{2x}$

x-intercepts:

$$0 = \frac{x^2 - 4}{2x}$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

y-intercepts:

$$y = \frac{0^2 - 4}{2(0)} = \frac{-4}{0} \quad \text{undefined}$$

The intercepts are $(-2, 0)$ and $(2, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$$-y = \frac{x^2 - 4}{2x} \quad \text{different}$$

Test *y*-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^2 - 4}{2(-x)}$$

$$y = -\frac{x^2 - 4}{2x} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{(-x)^2 - 4}{2(-x)}$$

$$-y = \frac{x^2 - 4}{-2x}$$

$$y = \frac{x^2 - 4}{2x} \quad \text{same}$$

Therefore, the graph has origin symmetry.

69. $y = \dfrac{-x^3}{x^2 - 9}$

x-intercepts:

$$0 = \frac{-x^3}{x^2 - 9}$$

$$-x^3 = 0$$

$$x = 0$$

y-intercepts:

$$y = \frac{-0^3}{0^2 - 9} = \frac{0}{-9} = 0$$

The only intercept is $(0, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test *y*-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$-y = \frac{x^3}{x^2 - 9}$$

$$y = \frac{-x^3}{x^2 - 9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

70. $y = \dfrac{x^4 + 1}{2x^5}$

x-intercepts:

$$0 = \frac{x^4 + 1}{2x^5}$$

$$x^4 = -1$$

no real solution

y-intercepts:

$$y = \frac{0^4 + 1}{2(0)^5} = \frac{1}{0} \quad \text{undefined}$$

There are no intercepts for the graph of this equation.

Test *x*-axis symmetry: Let $y = -y$

$$-y = \frac{x^4 + 1}{2x^5} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$y = \frac{x^4 + 1}{-2x^5} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

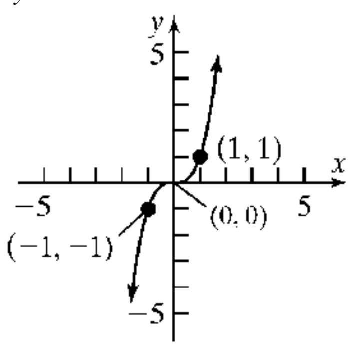
$$-y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$-y = \frac{x^4 + 1}{-2x^5}$$

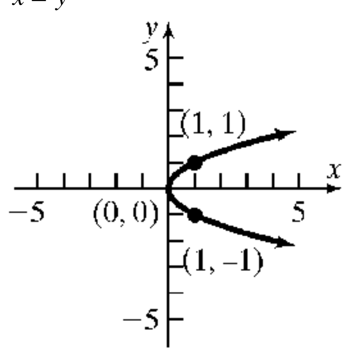
$$y = \frac{x^4 + 1}{2x^5} \quad \text{same}$$

Therefore, the graph has origin symmetry.

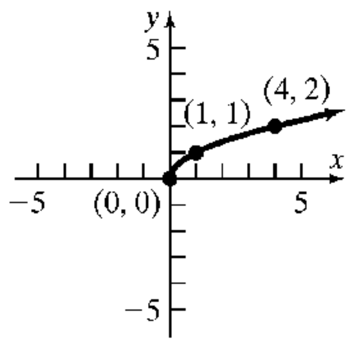
71. $y = x^3$



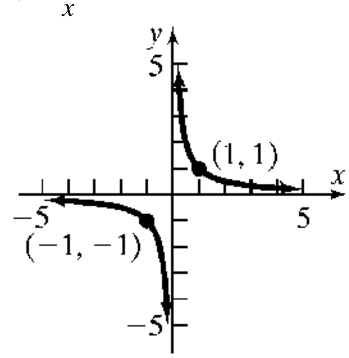
72. $x = y^2$



73. $y = \sqrt{x}$



74. $y = \frac{1}{x}$



75. If the point $(3, b)$ is on the graph of $y = 4x + 1$, then we have $b = 4(3) + 1 = 12 + 1 = 13$
Thus, $b = 13$.

76. If the point $(-2, b)$ is on the graph of $2x + 3y = 2$, then we have

$$2(-2) + 3(b) = 2$$
$$-4 + 3b = 2$$
$$3b = 6$$
$$b = 2$$

Thus, $b = 2$.

77. If the point $(a, 4)$ is on the graph of $y = x^2 + 3x$, then we have

$$4 = a^2 + 3a$$
$$0 = a^2 + 3a - 4$$
$$0 = (a + 4)(a - 1)$$
$$a + 4 = 0 \quad \text{or} \quad a - 1 = 0$$
$$a = -4 \qquad a = 1$$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a,-5)$ is on the graph of $y = x^2 + 6x$, then we have

$$-5 = a^2 + 6a$$
$$0 = a^2 + 6a + 5$$
$$0 = (a+5)(a+1)$$
$$a+5=0 \quad \text{or} \quad a+1=0$$
$$a=-5 \qquad\quad a=-1$$

Thus, $a=-5$ or $a=-1$.

79. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

80. For a graph with y-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an x-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the y-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another x-intercept.

81. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an x-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another x-intercept.

82. For a graph with x-axis symmetry, if the point (a,b) is on the graph, then so is the point $(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. a. $\left(x^2+y^2-x\right)^2 = x^2+y^2$

x-intercepts:

$$\left(x^2+(0)^2-x\right)^2 = x^2+(0)^2$$
$$\left(x^2-x\right)^2 = x^2$$
$$x^4-2x^3+x^2 = x^2$$
$$x^4-2x^3 = 0$$
$$x^3(x-2) = 0$$
$$x^3=0 \quad \text{or} \quad x-2=0$$
$$x=0 \qquad\quad x=2$$

y-intercepts:

$$\left((0)^2+y^2-0\right)^2 = (0)^2+y^2$$
$$\left(y^2\right)^2 = y^2$$
$$y^4 = y^2$$
$$y^4-y^2 = 0$$
$$y^2\left(y^2-1\right) = 0$$
$$y^2=0 \quad \text{or} \quad y^2-1=0$$
$$y=0 \qquad\quad y^2=1$$
$$y=\pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. <u>Test x-axis symmetry:</u> Let $y=-y$

$$\left(x^2+(-y)^2-x\right)^2 = x^2+(-y)^2$$
$$\left(x^2+y^2-x\right)^2 = x^2+y^2 \quad \text{same}$$

<u>Test y-axis symmetry:</u> Let $x=-x$

$$\left((-x)^2+y^2-(-x)\right)^2 = (-x)^2+y^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

<u>Test origin symmetry</u>: Let $x=-x$ and $y=-y$

$$\left((-x)^2+(-y)^2-(-x)\right)^2 = (-x)^2+(-y)^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

84. a. $16y^2 = 120x - 225$

x-intercepts:

$16y^2 = 120(0) - 225$

$16y^2 = -225$

$y^2 = -\dfrac{225}{16}$

no real solution

y-intercepts:

$16(0)^2 = 120x - 225$

$0 = 120x - 225$

$-120x = -225$

$x = \dfrac{-225}{-120} = \dfrac{15}{8}$

The only intercept is $\left(\dfrac{15}{8}, 0\right)$.

b. Test x-axis symmetry: Let $y = -y$

$16(-y)^2 = 120x - 225$

$16y^2 = 120x - 225$ same

Test y-axis symmetry: Let $x = -x$

$16y^2 = 120(-x) - 225$

$16y^2 = -120x - 225$ different

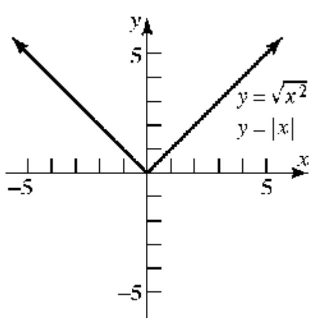
Test origin symmetry: Let $x = -x$ and $y = -y$

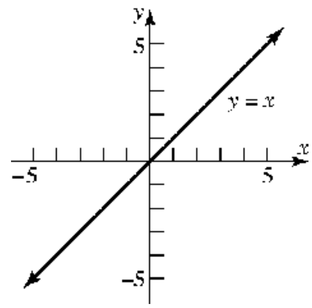
$16(-y)^2 = 120(-x) - 225$

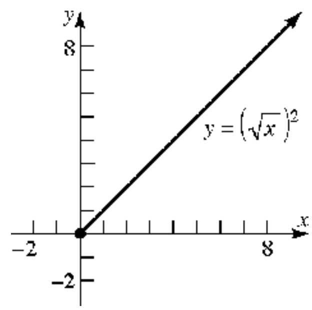
$16y^2 = -120x - 225$ different

Thus, the graph will have x-axis symmetry.

85. a.







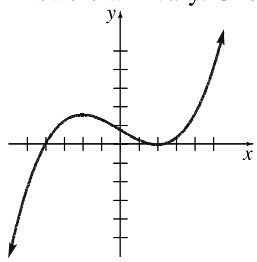
b. Since $\sqrt{x^2} = |x|$ for all x, the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

c. For $y = \left(\sqrt{x}\right)^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2 = x$ only for $x \geq 0$.

d. For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$. Otherwise, $\sqrt{x^2} = -x$.

86. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

87. Answers will vary. One example:



88. Answers will vary

89. Answers will vary

90. Answers will vary.
Case 1: Graph has x-axis and y-axis symmetry, show origin symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from x-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, -y)$ on graph
(from y-axis symmetry)
Since the point $(-x, -y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has x-axis and origin symmetry, show y-axis symmetry.
(x, y) on graph $\rightarrow (x, -y)$ on graph
(from x-axis symmetry)
$(x, -y)$ on graph $\rightarrow (-x, y)$ on graph
(from origin symmetry)
Since the point $(-x, y)$ is also on the graph, the graph has y-axis symmetry.

Case 3: Graph has y-axis and origin symmetry, show x-axis symmetry.
(x, y) on graph $\rightarrow (-x, y)$ on graph
(from y-axis symmetry)
$(-x, y)$ on graph $\rightarrow (x, -y)$ on graph
(from origin symmetry)
Since the point $(x, -y)$ is also on the graph, the graph has x-axis symmetry.

91. Answers may vary. The graph must contain the points $(-2, 5)$, $(-1, 3)$, and $(0, 2)$. For the graph to be symmetric about the y-axis, the graph must also contain the points $(2, 5)$ and $(1, 3)$ (note that (0, 2) is on the y-axis).

For the graph to also be symmetric with respect to the x-axis, the graph must also contain the points $(-2, -5)$, $(-1, -3)$, $(0, -2)$, $(2, -5)$, and $(1, -3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

92 – 94. Interactive Exercises

Section 2.3

1. undefined; 0

2. 3; 2
x-intercept: $2x + 3(0) = 6$
$$2x = 6$$
$$x = 3$$
y-intercept: $2(0) + 3y = 6$
$$3y = 6$$
$$y = 2$$

3. $y = b$; y-intercept

4. True

5. False; the slope is $\frac{3}{2}$.
$$2y = 3x + 5$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

6. True; $2(1) + (2) \stackrel{?}{=} 4$
$$2 + 2 \stackrel{?}{=} 4$$
$$4 = 4 \text{ True}$$

7. $m_1 = m_2$; y-intercepts; $m_1 \cdot m_2 = -1$

8. 2

9. $-\frac{1}{2}$

10. False; perpendicular lines have slopes that are opposite-reciprocals of each other.

11. **a.** Slope $= \dfrac{1-0}{2-0} = \dfrac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

12. **a.** Slope $= \dfrac{1-0}{-2-0} = -\dfrac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

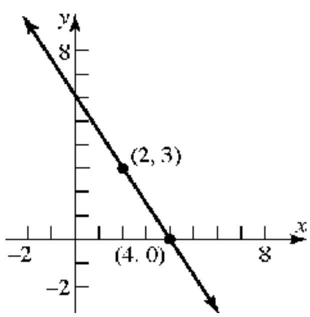
13. **a.** Slope $= \dfrac{1-2}{1-(-2)} = -\dfrac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

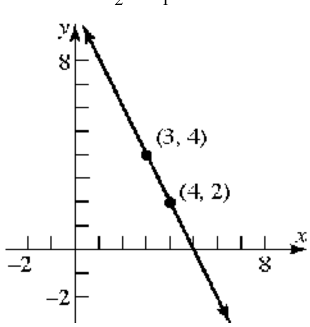
14. **a.** Slope $= \dfrac{2-1}{2-(-1)} = \dfrac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

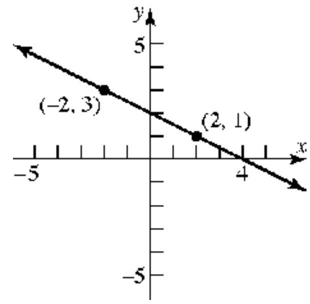
15. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-3}{4-2} = -\dfrac{3}{2}$



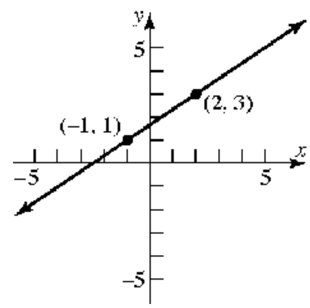
16. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-2}{3-4} = \dfrac{2}{-1} = -2$



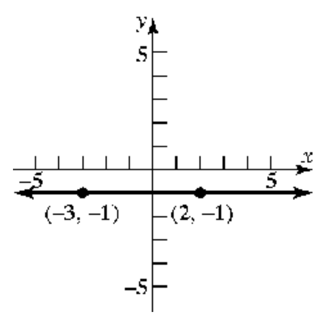
17. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



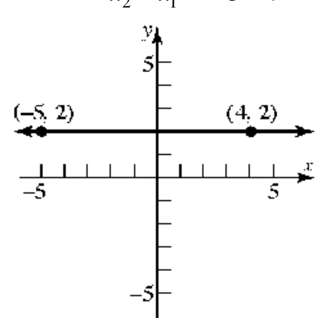
18. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



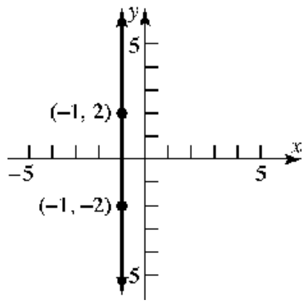
19. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



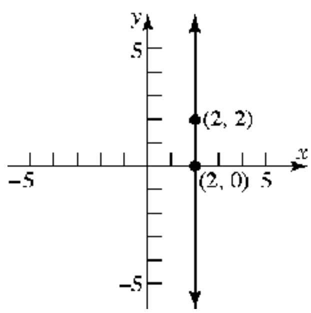
20. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



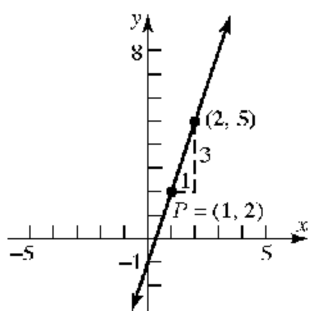
21. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



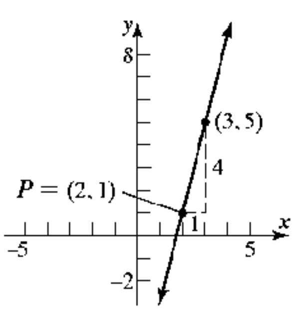
22. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



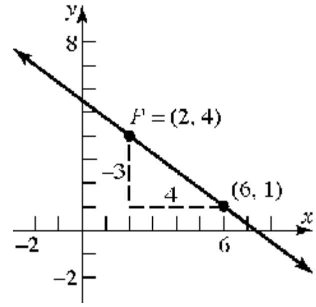
23. $P = (1,2); m = 3$



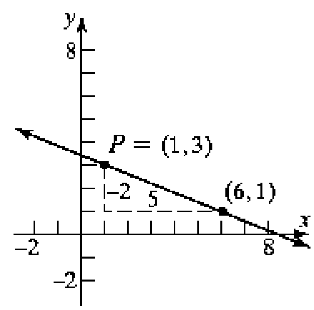
24. $P = (2,1); m = 4$



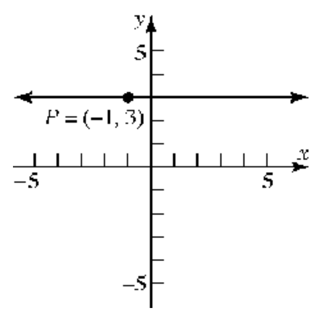
25. $P = (2,4); m = -\dfrac{3}{4}$



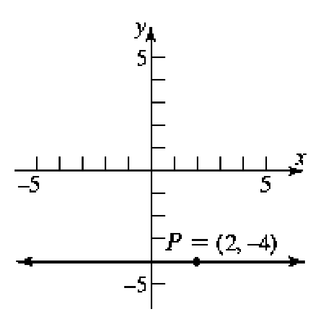
26. $P = (1,3); m = -\dfrac{2}{5}$



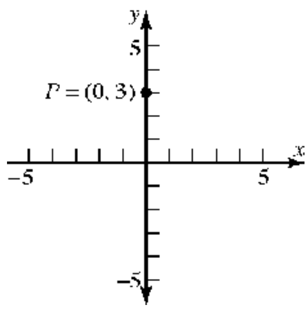
27. $P = (-1,3); m = 0$



28. $P = (2,-4); m = 0$

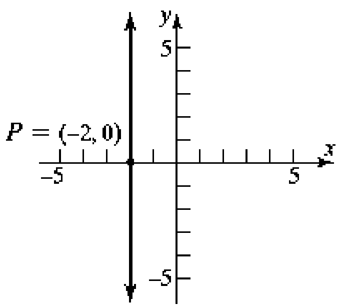


29. $P=(0,3)$; slope undefined



(note: the line is the *y*-axis)

30. $P=(-2,0)$; slope undefined



31. Slope $=4=\frac{4}{1}$; point: $(1,2)$

If *x* increases by 1 unit, then *y* increases by 4 units.
Answers will vary. Three possible points are:
$x=1+1=2$ and $y=2+4=6$
$(2,6)$
$x=2+1=3$ and $y=6+4=10$
$(3,10)$
$x=3+1=4$ and $y=10+4=14$
$(4,14)$

32. Slope $=2=\frac{2}{1}$; point: $(-2,3)$

If *x* increases by 1 unit, then *y* increases by 2 units.
Answers will vary. Three possible points are:
$x=-2+1=-1$ and $y=3+2=5$
$(-1,5)$
$x=-1+1=0$ and $y=5+2=7$
$(0,7)$
$x=0+1=1$ and $y=7+2=9$
$(1,9)$

33. Slope $=-\frac{3}{2}=\frac{-3}{2}$; point: $(2,-4)$

If *x* increases by 2 units, then *y* decreases by 3 units.
Answers will vary. Three possible points are:
$x=2+2=4$ and $y=-4-3=-7$
$(4,-7)$
$x=4+2=6$ and $y=-7-3=-10$
$(6,-10)$
$x=6+2=8$ and $y=-10-3=-13$
$(8,-13)$

34. Slope $=\frac{4}{3}$; point: $(-3,2)$

If *x* increases by 3 units, then *y* increases by 4 units.
Answers will vary. Three possible points are:
$x=-3+3=0$ and $y=2+4=6$
$(0,6)$
$x=0+3=3$ and $y=6+4=10$
$(3,10)$
$x=3+3=6$ and $y=10+4=14$
$(6,14)$

35. Slope $=-2=\frac{-2}{1}$; point: $(-2,-3)$

If *x* increases by 1 unit, then *y* decreases by 2 units.
Answers will vary. Three possible points are:
$x=-2+1=-1$ and $y=-3-2=-5$
$(-1,-5)$
$x=-1+1=0$ and $y=-5-2=-7$
$(0,-7)$
$x=0+1=1$ and $y=-7-2=-9$
$(1,-9)$

36. Slope $=-1=\frac{-1}{1}$; point: $(4,1)$

If *x* increases by 1 unit, then *y* decreases by 1 unit.

Answers will vary. Three possible points are:
$x = 4 + 1 = 5$ and $y = 1 - 1 = 0$
$(5, 0)$
$x = 5 + 1 = 6$ and $y = 0 - 1 = -1$
$(6, -1)$
$x = 6 + 1 = 7$ and $y = -1 - 1 = -2$
$(7, -2)$

37. (0, 0) and (2, 1) are points on the line.
$\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$
y-intercept is 0; using $y = mx + b$:
$$y = \frac{1}{2}x + 0$$
$$2y = x$$
$$0 = x - 2y$$
$x - 2y = 0$ or $y = \frac{1}{2}x$

38. (0, 0) and (–2, 1) are points on the line.
$\text{Slope} = \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$
y-intercept is 0; using $y = mx + b$:
$$y = -\frac{1}{2}x + 0$$
$$2y = -x$$
$$x + 2y = 0$$
$x + 2y = 0$ or $y = -\frac{1}{2}x$

39. (–1, 3) and (1, 1) are points on the line.
$\text{Slope} = \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$
Using $y - y_1 = m(x - x_1)$
$$y - 1 = -1(x - 1)$$
$$y - 1 = -x + 1$$
$$y = -x + 2$$
$x + y = 2$ or $y = -x + 2$

40. (–1, 1) and (2, 2) are points on the line.
$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$
Using $y - y_1 = m(x - x_1)$
$$y - 1 = \frac{1}{3}(x - (-1))$$
$$y - 1 = \frac{1}{3}(x + 1)$$
$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$
$$y = \frac{1}{3}x + \frac{4}{3}$$
$x - 3y = -4$ or $y = \frac{1}{3}x + \frac{4}{3}$

41. $y - y_1 = m(x - x_1)$, $m = 2$
$$y - 3 = 2(x - 3)$$
$$y - 3 = 2x - 6$$
$$y = 2x - 3$$
$2x - y = 3$ or $y = 2x - 3$

42. $y - y_1 = m(x - x_1)$, $m = -1$
$$y - 2 = -1(x - 1)$$
$$y - 2 = -x + 1$$
$$y = -x + 3$$
$x + y = 3$ or $y = -x + 3$

43. $y - y_1 = m(x - x_1)$, $m = -\frac{1}{2}$
$$y - 2 = -\frac{1}{2}(x - 1)$$
$$y - 2 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{5}{2}$$
$x + 2y = 5$ or $y = -\frac{1}{2}x + \frac{5}{2}$

44. $y - y_1 = m(x - x_1)$, $m = 1$
$$y - 1 = 1(x - (-1))$$
$$y - 1 = x + 1$$
$$y = x + 2$$
$x - y = -2$ or $y = x + 2$

45. Slope = 3; containing (–2, 3)

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 3(x - (-2))$$
$$y - 3 = 3x + 6$$
$$y = 3x + 9$$
$$3x - y = -9 \text{ or } y = 3x + 9$$

46. Slope = 2; containing the point (4, –3)

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 2(x - 4)$$
$$y + 3 = 2x - 8$$
$$y = 2x - 11$$
$$2x - y = 11 \text{ or } y = 2x - 11$$

47. Slope $= -\frac{2}{3}$; containing (1, –1)

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y + 1 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$
$$2x + 3y = -1 \text{ or } y = -\frac{2}{3}x - \frac{1}{3}$$

48. Slope $= \frac{1}{2}$; containing the point (3, 1)

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{2}(x - 3)$$
$$y - 1 = \frac{1}{2}x - \frac{3}{2}$$
$$y = \frac{1}{2}x - \frac{1}{2}$$
$$x - 2y = 1 \text{ or } y = \frac{1}{2}x - \frac{1}{2}$$

49. Containing (1, 3) and (–1, 2)

$$m = \frac{2-3}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{2}(x - 1)$$
$$y - 3 = \frac{1}{2}x - \frac{1}{2}$$
$$y = \frac{1}{2}x + \frac{5}{2}$$
$$x - 2y = -5 \text{ or } y = \frac{1}{2}x + \frac{5}{2}$$

50. Containing the points (–3, 4) and (2, 5)

$$m = \frac{5-4}{2-(-3)} = \frac{1}{5}$$
$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{1}{5}(x - 2)$$
$$y - 5 = \frac{1}{5}x - \frac{2}{5}$$
$$y = \frac{1}{5}x + \frac{23}{5}$$
$$x - 5y = -23 \text{ or } y = \frac{1}{5}x + \frac{23}{5}$$

51. Slope = –3; y-intercept =3

$$y = mx + b$$
$$y = -3x + 3$$
$$3x + y = 3 \text{ or } y = -3x + 3$$

52. Slope = –2; y-intercept = –2

$$y = mx + b$$
$$y = -2x + (-2)$$
$$2x + y = -2 \text{ or } y = -2x - 2$$

53. x-intercept = 2; y-intercept = –1
Points are (2,0) and (0,–1)

$$m = \frac{-1-0}{0-2} = \frac{-1}{-2} = \frac{1}{2}$$
$$y = mx + b$$
$$y = \frac{1}{2}x - 1$$
$$x - 2y = 2 \text{ or } y = \frac{1}{2}x - 1$$

54. x-intercept = –4; y-intercept = 4
Points are (–4, 0) and (0, 4)
$$m = \frac{4-0}{0-(-4)} = \frac{4}{4} = 1$$
$$y = mx + b$$
$$y = 1x + 4$$
$$y = x + 4$$
$$x - y = -4 \text{ or } y = x + 4$$

55. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x = 2$ No slope-intercept form.

56. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x = 3$ No slope-intercept form.

57. Horizontal lines have slope $m = 0$ and take the form $y = b$. Therefore, the horizontal line passing through the point $(-3, 2)$ is $y = 2$.

58. Vertical lines have an undefined slope and take the form $x = a$. Therefore, the vertical line passing through the point $(4, -5)$ is $x = 4$.

59. Parallel to $y = 2x$; Slope = 2
Containing (–1, 2)
$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - (-1))$$
$$y - 2 = 2x + 2 \rightarrow y = 2x + 4$$
$$2x - y = -4 \text{ or } y = 2x + 4$$

60. Parallel to $y = -3x$; Slope = –3; Containing the point (–1, 2)
$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - (-1))$$
$$y - 2 = -3x - 3 \rightarrow y = -3x - 1$$
$$3x + y = -1 \text{ or } y = -3x - 1$$

61. Parallel to $2x - y = -2$; Slope = 2
Containing the point (0, 0)
$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$2x - y = 0 \text{ or } y = 2x$$

62. Parallel to $x - 2y = -5$;
Slope = $\frac{1}{2}$; Containing the point $(0, 0)$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0) \rightarrow y = \frac{1}{2}x$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

63. Parallel to $x = 5$; Containing (4,2)
This is a vertical line.
$x = 4$ No slope-intercept form.

64. Parallel to $y = 5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y = 2$

65. Perpendicular to $y = \frac{1}{2}x + 4$; Containing (1, –2)
Slope of perpendicular = –2
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 1)$$
$$y + 2 = -2x + 2 \rightarrow y = -2x$$
$$2x + y = 0 \text{ or } y = -2x$$

66. Perpendicular to $y = 2x - 3$; Containing the point (1, –2)
Slope of perpendicular = $-\frac{1}{2}$
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$y + 2 = -\frac{1}{2}x + \frac{1}{2} \rightarrow y = -\frac{1}{2}x - \frac{3}{2}$$
$$x + 2y = -3 \text{ or } y = -\frac{1}{2}x - \frac{3}{2}$$

67. Perpendicular to $2x+y=2$; Containing the point (–3, 0)

Slope of perpendicular $=\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-0=\frac{1}{2}(x-(-3)) \rightarrow y=\frac{1}{2}x+\frac{3}{2}$

$x-2y=-3$ or $y=\frac{1}{2}x+\frac{3}{2}$

68. Perpendicular to $x-2y=-5$; Containing the point (0, 4)

Slope of perpendicular = –2

$y=mx+b$

$y=-2x+4$

$2x+y=4$ or $y=-2x+4$

69. Perpendicular to $x=8$; Containing (3, 4)

Slope of perpendicular = 0 (horizontal line)

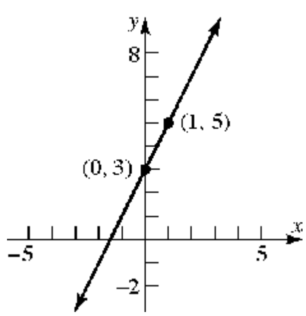
$y=4$

70. Perpendicular to $y=8$;

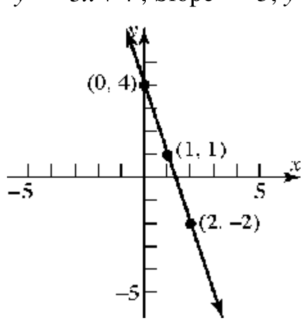
Containing the point (3, 4)

Slope of perpendicular is undefined (vertical line). $x=3$ No slope-intercept form.

71. $y=2x+3$; Slope = 2; y-intercept = 3

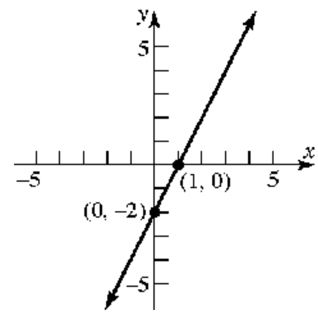


72. $y=-3x+4$; Slope = –3; y-intercept = 4



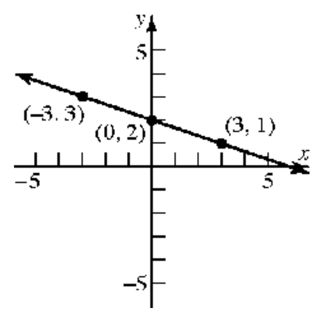
73. $\frac{1}{2}y=x-1$; $y=2x-2$

Slope = 2; y-intercept = –2

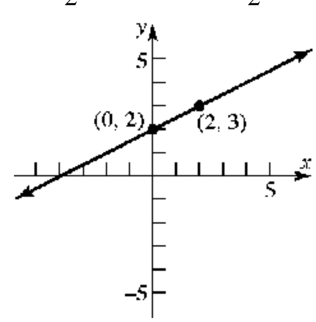


74. $\frac{1}{3}x+y=2$; $y=-\frac{1}{3}x+2$

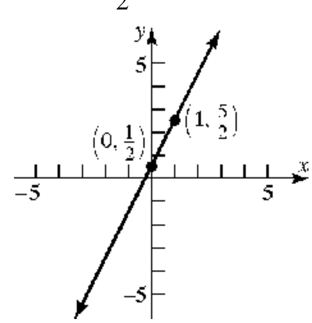
Slope $=-\frac{1}{3}$; y-intercept = 2



75. $y=\frac{1}{2}x+2$; Slope $=\frac{1}{2}$; y-intercept = 2

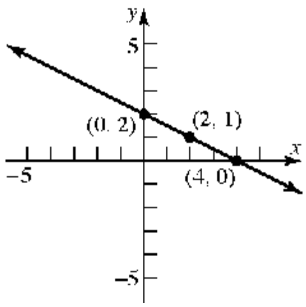


76. $y=2x+\frac{1}{2}$; Slope = 2; y-intercept $=\frac{1}{2}$



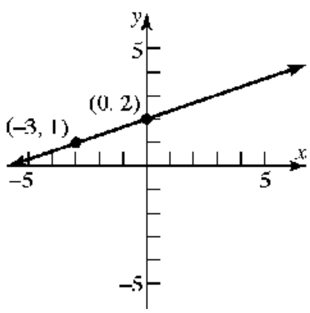
77. $x+2y=4$; $2y=-x+4 \to y=-\frac{1}{2}x+2$

Slope $=-\frac{1}{2}$; y-intercept = 2



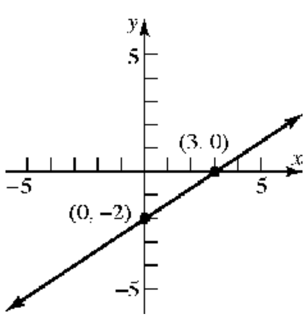
78. $-x+3y=6$; $3y=x+6 \to y=\frac{1}{3}x+2$

Slope $=\frac{1}{3}$; y-intercept = 2



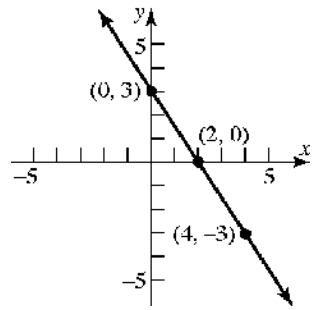
79. $2x-3y=6$; $-3y=-2x+6 \to y=\frac{2}{3}x-2$

Slope $=\frac{2}{3}$; y-intercept = –2

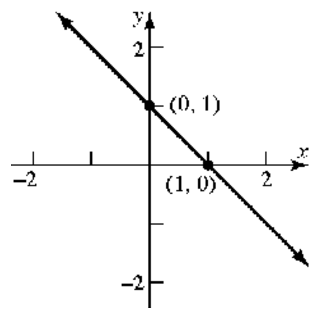


80. $3x+2y=6$; $2y=-3x+6 \to y=-\frac{3}{2}x+3$

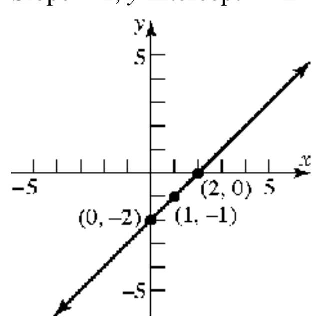
Slope $=-\frac{3}{2}$; y-intercept = 3



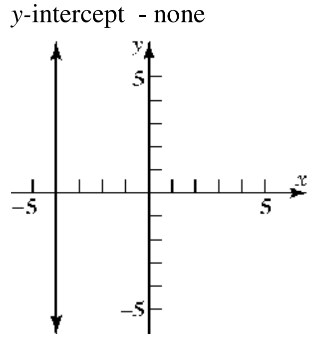
81. $x+y=1$; $y=-x+1$
Slope = –1; y-intercept = 1



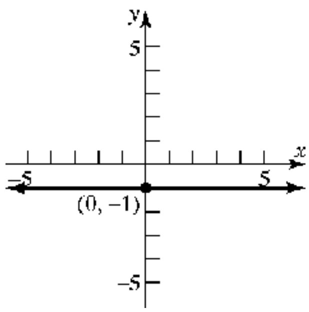
82. $x-y=2$; $y=x-2$
Slope = 1; y-intercept = –2



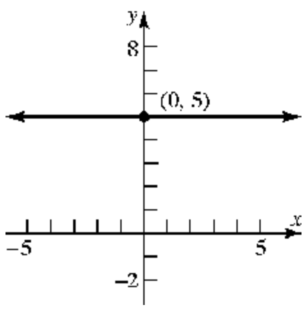
83. $x=-4$; Slope is undefined
y-intercept - none



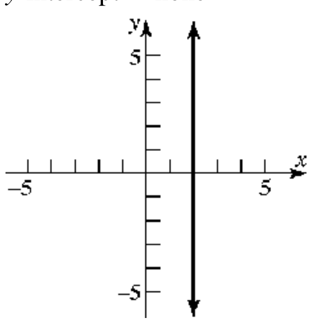
84. $y = -1$; Slope = 0; y-intercept $= -1$



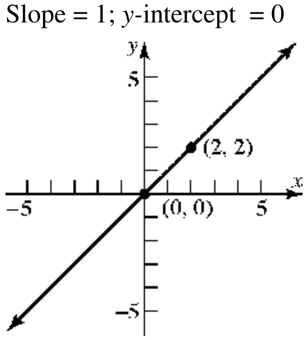
85. $y = 5$; Slope = 0; y-intercept = 5



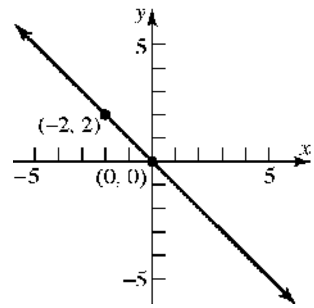
86. $x = 2$; Slope is undefined
y-intercept - none



87. $y - x = 0$; $y = x$
Slope = 1; y-intercept $= 0$

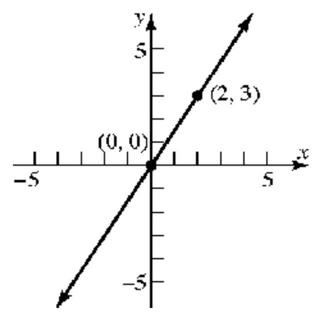


88. $x + y = 0$; $y = -x$
Slope $= -1$; y-intercept $= 0$



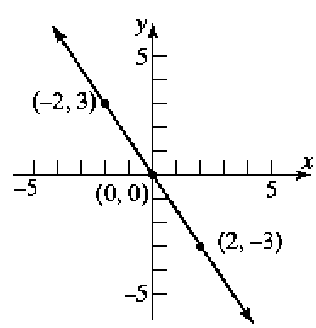
89. $2y - 3x = 0$; $2y = 3x \rightarrow y = \frac{3}{2}x$

Slope $= \frac{3}{2}$; y-intercept = 0



90. $3x + 2y = 0$; $2y = -3x \rightarrow y = -\frac{3}{2}x$

Slope $= -\frac{3}{2}$; y-intercept = 0



91. **a.** x-intercept: $2x+3(0)=6$

$$2x=6$$
$$x=3$$

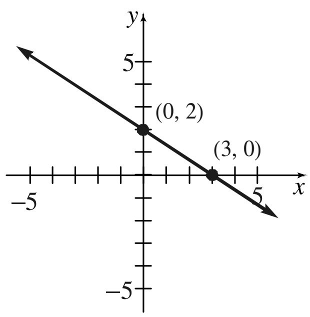
The point $(3,0)$ is on the graph.

y-intercept: $2(0)+3y=6$

$$3y=6$$
$$y=2$$

The point $(0,2)$ is on the graph.

b.



92. **a.** x-intercept: $3x-2(0)=6$

$$3x=6$$
$$x=2$$

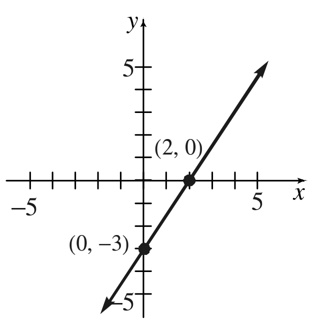
The point $(2,0)$ is on the graph.

y-intercept: $3(0)-2y=6$

$$-2y=6$$
$$y=-3$$

The point $(0,-3)$ is on the graph.

b.



93. **a.** x-intercept: $-4x+5(0)=40$

$$-4x=40$$
$$x=-10$$

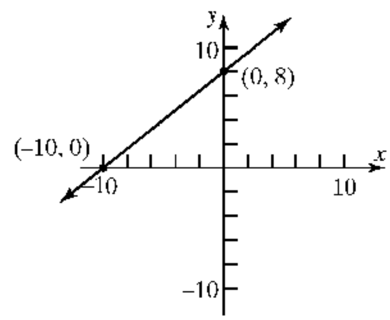
The point $(-10,0)$ is on the graph.

y-intercept: $-4(0)+5y=40$

$$5y=40$$
$$y=8$$

The point $(0,8)$ is on the graph.

b.



94. **a.** x-intercept: $6x-4(0)=24$

$$6x=24$$
$$x=4$$

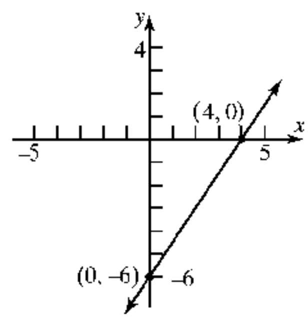
The point $(4,0)$ is on the graph.

y-intercept: $6(0)-4y=24$

$$-4y=24$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



95. **a.** x-intercept: $7x + 2(0) = 21$

$$7x = 21$$
$$x = 3$$

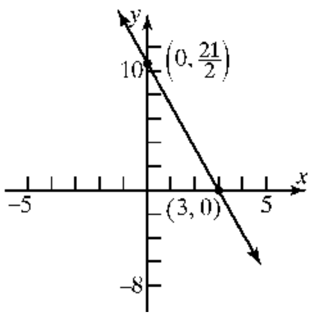
The point $(3,0)$ is on the graph.

y-intercept: $7(0) + 2y = 21$

$$2y = 21$$
$$y = \frac{21}{2}$$

The point $\left(0, \frac{21}{2}\right)$ is on the graph.

b.



96. **a.** x-intercept: $5x + 3(0) = 18$

$$5x = 18$$
$$x = \frac{18}{5}$$

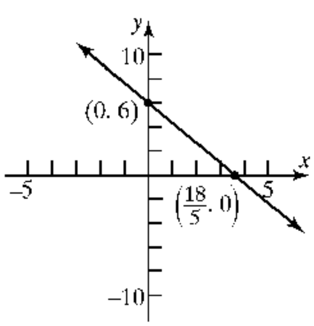
The point $\left(\frac{18}{5}, 0\right)$ is on the graph.

y-intercept: $5(0) + 3y = 18$

$$3y = 18$$
$$y = 6$$

The point $(0,6)$ is on the graph.

b.



97. **a.** x-intercept: $\frac{1}{2}x + \frac{1}{3}(0) = 1$

$$\frac{1}{2}x = 1$$
$$x = 2$$

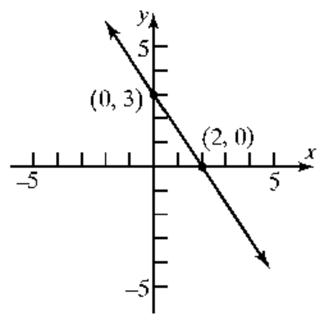
The point $(2,0)$ is on the graph.

y-intercept: $\frac{1}{2}(0) + \frac{1}{3}y = 1$

$$\frac{1}{3}y = 1$$
$$y = 3$$

The point $(0,3)$ is on the graph.

b.



98. **a.** x-intercept: $x - \frac{2}{3}(0) = 4$

$$x = 4$$

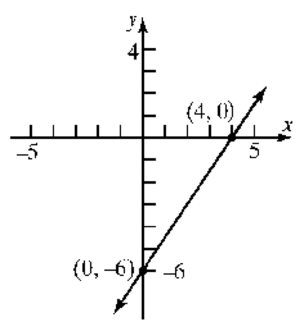
The point $(4,0)$ is on the graph.

y-intercept: $(0) - \frac{2}{3}y = 4$

$$-\frac{2}{3}y = 4$$
$$y = -6$$

The point $(0,-6)$ is on the graph.

b.



99. a. x-intercept: $0.2x - 0.5(0) = 1$

$$0.2x = 1$$
$$x = 5$$

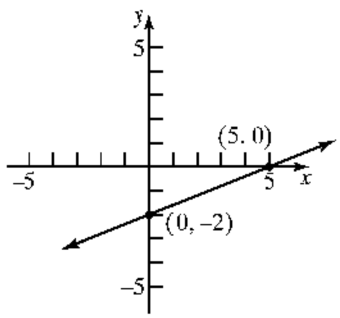
The point $(5, 0)$ is on the graph.

y-intercept: $0.2(0) - 0.5y = 1$

$$-0.5y = 1$$
$$y = -2$$

The point $(0, -2)$ is on the graph.

b.



100. a. x-intercept: $-0.3x + 0.4(0) = 1.2$

$$-0.3x = 1.2$$
$$x = -4$$

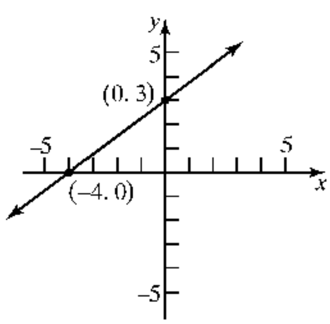
The point $(-4, 0)$ is on the graph.

y-intercept: $-0.3(0) + 0.4y = 1.2$

$$0.4y = 1.2$$
$$y = 3$$

The point $(0, 3)$ is on the graph.

b.



101. The equation of the x-axis is $y = 0$. (The slope is 0 and the y-intercept is 0.)

102. The equation of the y-axis is $x = 0$. (The slope is undefined.)

103. The slopes are the same but the y-intercepts are different. Therefore, the two lines are parallel.

104. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

105. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

106. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

107. Intercepts: $(0, 2)$ and $(-2, 0)$. Thus, slope = 1.

$y = x + 2$ or $x - y = -2$

108. Intercepts: $(0, 1)$ and $(1, 0)$. Thus, slope = –1.

$y = -x + 1$ or $x + y = 1$

109. Intercepts: $(3, 0)$ and $(0, 1)$. Thus, slope = $-\frac{1}{3}$.

$y = -\frac{1}{3}x + 1$ or $x + 3y = 3$

110. Intercepts: $(0, -1)$ and $(-2, 0)$. Thus,

slope = $-\frac{1}{2}$.

$y = -\frac{1}{2}x - 1$ or $x + 2y = -2$

111. $P_1 = (-2, 5)$, $P_2 = (1, 3)$: $m_1 = \frac{5-3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$

$P_2 = (1, 3)$, $P_3 = (-1, 0)$: $m_2 = \frac{3-0}{1-(-1)} = \frac{3}{2}$

Since $m_1 \cdot m_2 = -1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

112. $P_1 = (1, -1)$, $P_2 = (4, 1)$, $P_3 = (2, 2)$, $P_4 = (5, 4)$

$m_{12} = \frac{1-(-1)}{4-1} = \frac{2}{3}$; $m_{24} = \frac{4-1}{5-4} = 3$;

$m_{34} = \frac{4-2}{5-2} = \frac{2}{3}$; $m_{13} = \frac{2-(-1)}{2-1} = 3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

113. $P_1 = (-1,0)$, $P_2 = (2,3)$, $P_3 = (1,-2)$, $P_4 = (4,1)$

$m_{12} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1$; $m_{24} = \frac{1-3}{4-2} = -1$;

$m_{34} = \frac{1-(-2)}{4-1} = \frac{3}{3} = 1$; $m_{13} = \frac{-2-0}{1-(-1)} = -1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

114. $P_1 = (0,0)$, $P_2 = (1,3)$, $P_3 = (4,2)$, $P_4 = (3,-1)$

$m_{12} = \frac{3-0}{1-0} = 3$; $m_{23} = \frac{2-3}{4-1} = -\frac{1}{3}$;

$m_{34} = \frac{-1-2}{3-4} = 3$; $m_{14} = \frac{-1-0}{3-0} = -\frac{1}{3}$

$d_{12} = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$

$d_{23} = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$d_{34} = \sqrt{(3-4)^2 + (-1-2)^2} = \sqrt{1+9} = \sqrt{10}$

$d_{14} = \sqrt{(3-0)^2 + (-1-0)^2} = \sqrt{9+1} = \sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

115. Let x = number of miles driven, and let C = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C = 0.20x + 29$
When x = 110, $C = (0.20)(110) + 29 = \$51.00$.
When x = 230, $C = (0.20)(230) + 29 = \$75.00$.

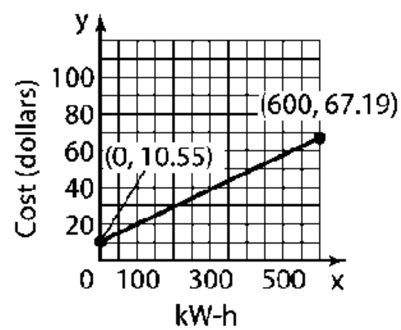
116. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C = 8x + 500$
When x = 400, $C = (8)(400) + 500 = \$3700$.
When x = 740, $C = (8)(740) + 500 = \$6420$.

117. Let x = number of miles driven annually, and let C = cost in dollars.
Total cost = (approx cost per mile)(number of miles) + fixed cost
$C = 0.15x + 1289$

118. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S = 0.05x + 375$

119. **a.** $C = 0.0944x + 10.55$; $0 \le x \le 600$

b.



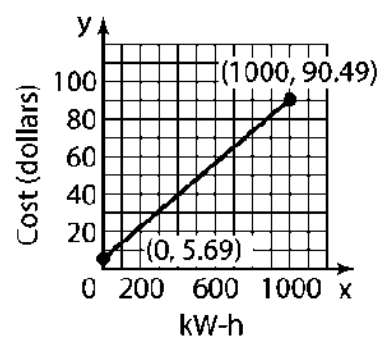
c. For 200 kWh,
$C = 0.0944(200) + 10.55 = \$29.43$

d. For 500 kWh,
$C = 0.0944(500) + 10.55 = \$57.75$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.0944 (that is, 9.44 cents).

120. **a.** $C = 0.0848x + 5.69$; $0 \le x \le 1000$

b.



c. For 200 kWh,
$C = 0.0848(200) + 5.69 = \$22.65$

d. For 500 kWh,
$C = 0.0848(500) + 5.69 = \$48.09$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.0848 (that is, 8.48 cents).

121. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 70$, then

$$°C = \frac{5}{9}(70-32) = \frac{5}{9}(38)$$

$$°C \approx 21.1°$$

122. **a.** $K = °C + 273$

b. $°C = \frac{5}{9}(°F - 32)$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

$$K = \frac{1}{9}(5°F + 2297)$$

123. **a.** The y-intercept is (0, 30), so $b = 30$. Since the ramp drops 2 inches for every 25 inches of run, the slope is $m = \frac{-2}{25} = -\frac{2}{25}$. Thus, the equation is $y = -\frac{2}{25}x + 30$.

b. Let $y = 0$.

$$0 = -\frac{2}{25}x + 30$$

$$\frac{2}{25}x = 30$$

$$\frac{25}{2}\left(\frac{2}{25}x\right) = \frac{25}{2}(30)$$

$$x = 375$$

The x-intercept is (375, 0). This means that the ramp meets the floor 375 inches (or 31.25 feet) from the base of the platform.

c. No. From part (b), the run is 31.25 feet which exceeds the required maximum of 30 feet.

d. First, design requirements state that the maximum slope is a drop of 1 inch for each 12 inches of run. This means $|m| \le \frac{1}{12}$. Second, the run is restricted to be no more than 30 feet = 360 inches. For a rise of 30 inches, this means the minimum slope is $\frac{30}{360} = \frac{1}{12}$. That is, $|m| \ge \frac{1}{12}$. Thus, the only possible slope is $|m| = \frac{1}{12}$. The diagram indicates that the slope is negative. Therefore, the only slope that can be used to obtain the 30-inch rise and still meet design requirements is $m = -\frac{1}{12}$. In words, for every 12 inches of run, the ramp must drop *exactly* 1 inch.

124. **a.** The year 1996 corresponds to $x = 0$, and the year 2006 corresponds to $x = 10$. Therefore, the points (0, 22.2) and (7, 12.2) are on the line. Thus, $m = \frac{10-0}{12.2-22.2} = -\frac{10}{10} = -1$. The y-intercept is 22.2, so $b = 22.2$ and the equation is $y = -x + 22.2$

b. x-intercept: $0 = -x + 22.2$

$$x = 22.2$$

y-intercept: $y = -(0) + 22.2 = 22.2$

The intercepts are (22.2, 0) and (0, 22.2).

c. The y-intercept represents the percentage of twelfth graders in 1996 who had reported daily use of cigarettes. The x-intercept represents the number of years after 1996 when 0% of twelfth graders will have reported daily use of cigarettes.

d. The year 2016 corresponds to $x = 20$.

$$y = -(20) + 22.2 = 2.2$$

This prediction is reasonable.

125. **a.** Let x = number of boxes to be sold, and A = money, in dollars, spent on advertising. We have the points

$(x_1, A_1) = (100{,}000, 40{,}000)$;

$(x_2, A_2) = (200,000, 60,000)$

$$\text{slope} = \frac{60,000 - 40,000}{200,000 - 100,000} = \frac{20,000}{100,000} = \frac{1}{5}$$

$$A - 40,000 = \frac{1}{5}(x - 100,000)$$

$$A - 40,000 = \frac{1}{5}x - 20,000$$

$$A = \frac{1}{5}x + 20,000$$

b. If x = 300,000, then

$$A = \frac{1}{5}(300,000) + 20,000 = \$80,000$$

c. Each additional box sold requires an additional $0.20 in advertising.

126. Find the slope of the line containing (a,b) and (b,a):

$$\text{slope} = \frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

Since $-1 \cdot 1 = -1$, the line containing the points (a,b) and (b,a) is perpendicular to the line $y = x$.

The midpoint of (a,b) and (b,a) is

$$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right).$$

Since the coordinates are the same, the midpoint lies on the line $y = x$.

Note: $\dfrac{a+b}{2} = \dfrac{b+a}{2}$

127. $2x - y = C$

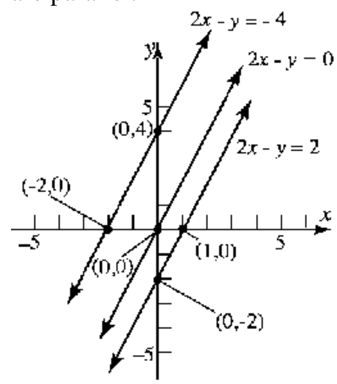
Graph the lines:

$2x - y = -4$

$2x - y = 0$

$2x - y = 2$

All the lines have the same slope, 2. The lines are parallel.



128. Refer to Figure 47.

length of $\overline{OA} = d(O,A) = \sqrt{1 + m_1^2}$

length of $\overline{OB} = d(O,B) = \sqrt{1 + m_2^2}$

length of $\overline{AB} = d(A,B) = m_1 - m_2$

Now consider the equation

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

If this equation is valid, then ΔAOB is a right triangle with right angle at vertex *O*.

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

$$1 + m_1^2 + 1 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

But we are assuming that $m_1m_2 = -1$, so we have

$$2 + m_1^2 + m_2^2 = m_1^2 - 2(-1) + m_2^2$$

$$2 + m_1^2 + m_2^2 = m_1^2 + 2 + m_2^2$$

$$0 = 0$$

Therefore, by the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex *O*. Thus Line 1 is perpendicular to Line 2.

129. (b), (c), (e) and (g)
The line has positive slope and positive *y*-intercept.

130. (a), (c), and (g)
The line has negative slope and positive *y*-intercept.

131. (c)
The equation $x - y = -2$ has slope 1 and y-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and y-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive y-intercept and the other with a negative y-intercept.

132. (d)
The equation $y - 2x = 2$ has slope 2 and y-intercept $(0, 2)$. The equation $x + 2y = -1$ has slope $-\frac{1}{2}$ and y-intercept $\left(0, -\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right) = -1$. One line has a positive y-intercept and the other with a negative y-intercept.

133 – 135. Answers will vary.

136. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

137. No, a line does not need to have both an x-intercept and a y-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

138. Two lines with equal slopes and equal y-intercepts are coinciding lines (i.e. the same).

139. Two lines that have the same x-intercept and y-intercept (assuming the x-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

140. No. Two lines with the same slope and different x-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,
Line 1 has x-intercept $-\frac{b_1}{m}$ and y-intercept b_1.
Line 2 has x-intercept $-\frac{b_2}{m}$ and y-intercept b_2.
Assume also that Line 1 and Line 2 have unequal x-intercepts.
If the lines have the same y-intercept, then $b_1 = b_2$.

$$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

141. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.
Assume Line 1 has equation $y = m_1 x + b$ and Line 2 has equation $y = m_2 x + b$,
Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.
Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.
Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.
If the lines have the same x-intercept, then $-\frac{b}{m_1} = -\frac{b}{m_2}$.

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$
$$-m_2 b = -m_1 b$$
$$-m_2 b + m_1 b = 0$$

But $-m_2 b + m_1 b = 0 \Rightarrow b(m_1 - m_2) = 0$
$\Rightarrow b = 0$
or $m_1 - m_2 = 0 \Rightarrow m_1 = m_2$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

142. Answers will vary.

143. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{1 - (-3)} = \frac{-6}{4} = -\frac{3}{2}$
It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

144. Interactive Exercise

Section 2.4

1. add; $\left(\frac{1}{2}\cdot 10\right)^2 = 25$

2. $(x-2)^2 = 9$
   $x-2 = \pm\sqrt{9}$
   $x-2 = \pm 3$
   $x = 2 \pm 3$
   $x = 5 \quad \text{or} \quad x = -1$
   The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

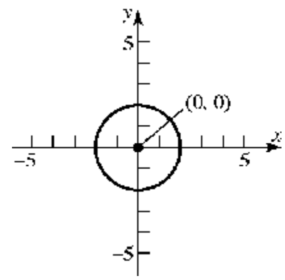
7. Center = (2, 1)
   Radius = distance from (0,1) to (2,1)
   $= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$
   Equation: $(x-2)^2 + (y-1)^2 = 4$

8. Center = (1, 2)
   Radius = distance from (1,0) to (1,2)
   $= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$
   Equation: $(x-1)^2 + (y-2)^2 = 4$

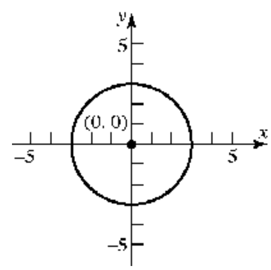
9. Center = midpoint of (1, 2) and (4, 2)
   $= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$
   Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)
   $= \sqrt{\left(4-\frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$
   Equation: $\left(x-\frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

10. Center = midpoint of (0, 1) and (2, 3)
   $= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$
   Radius = distance from $(1,2)$ to (2,3)
   $= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$
   Equation: $(x-1)^2 + (y-2)^2 = 2$

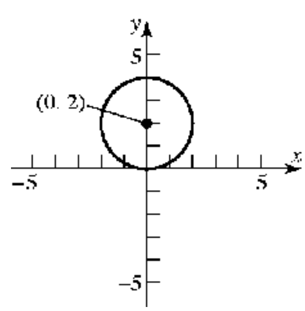
11. $(x-h)^2 + (y-k)^2 = r^2$
   $(x-0)^2 + (y-0)^2 = 2^2$
   $x^2 + y^2 = 4$
   General form: $x^2 + y^2 - 4 = 0$



12. $(x-h)^2 + (y-k)^2 = r^2$
   $(x-0)^2 + (y-0)^2 = 3^2$
   $x^2 + y^2 = 9$
   General form: $x^2 + y^2 - 9 = 0$



13. $(x-h)^2 + (y-k)^2 = r^2$
   $(x-0)^2 + (y-2)^2 = 2^2$
   $x^2 + (y-2)^2 = 4$
   General form: $x^2 + y^2 - 4y + 4 = 4$
   $x^2 + y^2 - 4y = 0$



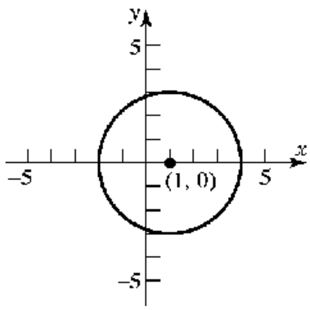
14. $(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-0)^2=3^2$

$(x-1)^2+y^2=9$

General form: $x^2-2x+1+y^2=9$

$x^2+y^2-2x-8=0$



15. $(x-h)^2+(y-k)^2=r^2$

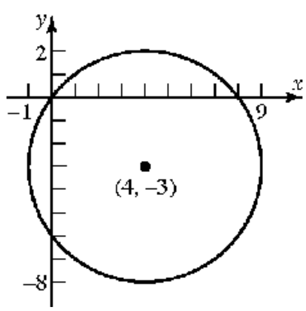
$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$



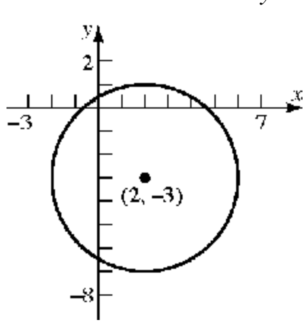
16. $(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form: $x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$



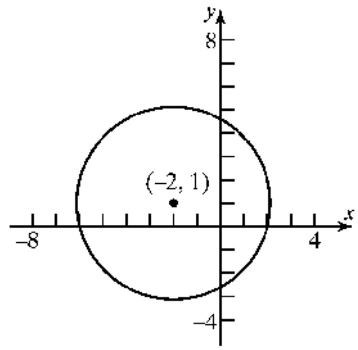
17. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



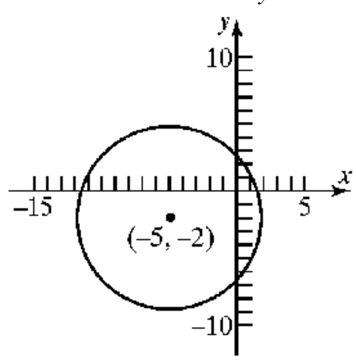
18. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



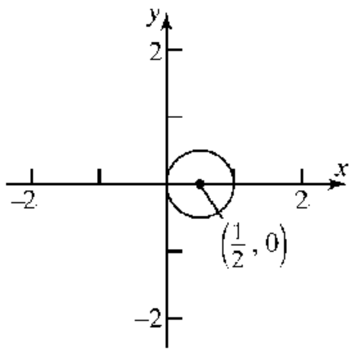
19. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{2}\right)^2+(y-0)^2=\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

General form: $x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

$x^2+y^2-x=0$



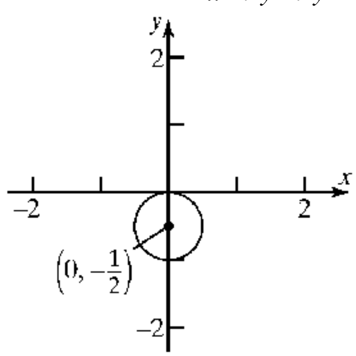
20. $(x-h)^2+(y-k)^2=r^2$

$$(x-0)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{1}{2}\right)^2$$

$$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$$

General form: $x^2+y^2+y+\frac{1}{4}=\frac{1}{4}$

$$x^2+y^2+y=0$$

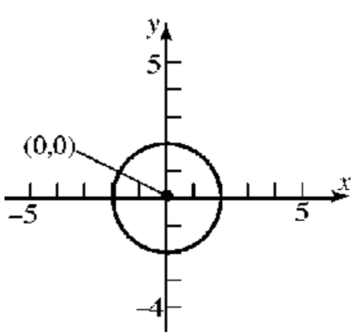


21. $x^2+y^2=4$

$x^2+y^2=2^2$

a. Center: $(0,0)$; Radius $=2$

b.



c. x-intercepts: $x^2+(0)^2=4$

$$x^2=4$$

$$x=\pm\sqrt{4}=\pm2$$

y-intercepts: $(0)^2+y^2=4$

$$y^2=4$$

$$y=\pm\sqrt{4}=\pm2$$

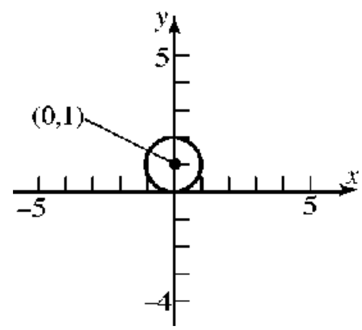
The intercepts are $(-2,0)$, $(2,0)$, $(0,-2)$, and $(0,2)$.

22. $x^2+(y-1)^2=1$

$x^2+(y-1)^2=1^2$

a. Center: $(0,1)$; Radius $=1$

b.



c. x-intercepts: $x^2+(0-1)^2=1$

$$x^2+1=1$$

$$x^2=0$$

$$x=\pm\sqrt{0}=0$$

y-intercepts: $(0)^2+(y-1)^2=1$

$$(y-1)^2=1$$

$$y-1=\pm\sqrt{1}$$

$$y-1=\pm1$$

$$y=1\pm1$$

$$y=2 \quad \text{or} \quad y=0$$

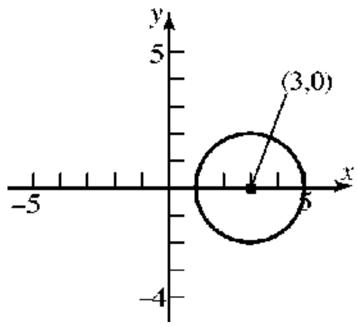
The intercepts are $(0,0)$ and $(0,2)$.

23. $2(x-3)^2+2y^2=8$

$(x-3)^2+y^2=4$

a. Center: $(3,0)$; Radius $=2$

b.



c. x-intercepts: $(x-3)^2+(0)^2=4$

$$(x-3)^2=4$$
$$x-3=\pm\sqrt{4}$$
$$x-3=\pm 2$$
$$x=3\pm 2$$
$$x=5 \quad \text{or} \quad x=1$$

y-intercepts: $(0-3)^2+y^2=4$

$$(-3)^2+y^2=4$$
$$9+y^2=4$$
$$y^2=-5$$

No real solution.

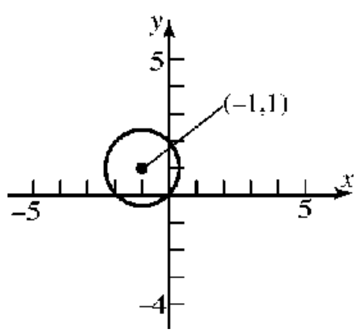
The intercepts are $(1,0)$ and $(5,0)$.

24. $3(x+1)^2+3(y-1)^2=6$

$$(x+1)^2+(y-1)^2=2$$

a. Center: (–1,1); Radius = $\sqrt{2}$

b.



c. x-intercepts: $(x+1)^2+(0-1)^2=2$

$$(x+1)^2+(-1)^2=2$$
$$(x+1)^2+1=2$$
$$(x+1)^2=1$$
$$x+1=\pm\sqrt{1}$$
$$x+1=\pm 1$$
$$x=-1\pm 1$$
$$x=0 \quad \text{or} \quad x=-2$$

y-intercepts: $(0+1)^2+(y-1)^2=2$

$$(1)^2+(y-1)^2=2$$
$$1+(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\pm\sqrt{1}$$
$$y-1=\pm 1$$
$$y=1\pm 1$$
$$y=2 \quad \text{or} \quad y=0$$

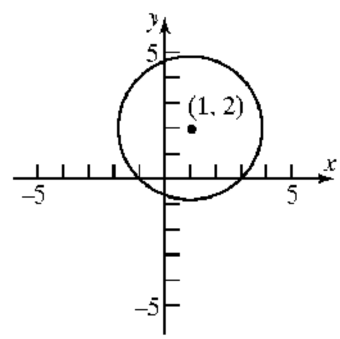
The intercepts are $(-2,0)$, $(0,0)$, and $(0,2)$.

25. $x^2+y^2-2x-4y-4=0$

$$x^2-2x+y^2-4y=4$$
$$(x^2-2x+1)+(y^2-4y+4)=4+1+4$$
$$(x-1)^2+(y-2)^2=3^2$$

a. Center: (1, 2); Radius = 3

b.



c. x-intercepts: $(x-1)^2+(0-2)^2=3^2$

$$(x-1)^2+(-2)^2=3^2$$
$$(x-1)^2+4=9$$
$$(x-1)^2=5$$
$$x-1=\pm\sqrt{5}$$
$$x=1\pm\sqrt{5}$$

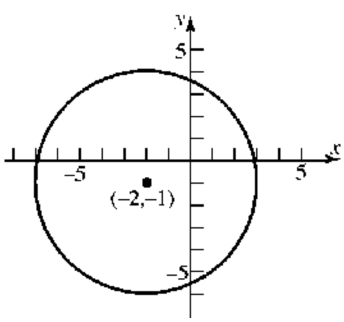
y-intercepts: $(0-1)^2+(y-2)^2=3^2$
$$(-1)^2+(y-2)^2=3^2$$
$$1+(y-2)^2=9$$
$$(y-2)^2=8$$
$$y-2=\pm\sqrt{8}$$
$$y-2=\pm 2\sqrt{2}$$
$$y=2\pm 2\sqrt{2}$$

The intercepts are $\left(1-\sqrt{5},0\right)$, $\left(1+\sqrt{5},0\right)$, $\left(0,2-2\sqrt{2}\right)$, and $\left(0,2+2\sqrt{2}\right)$.

26.
$$x^2+y^2+4x+2y-20=0$$
$$x^2+4x+y^2+2y=20$$
$$(x^2+4x+4)+(y^2+2y+1)=20+4+1$$
$$(x+2)^2+(y+1)^2=5^2$$

a. Center: (–2,–1); Radius = 5

b.



c. x-intercepts: $(x+2)^2+(0+1)^2=5^2$
$$(x+2)^2+1=25$$
$$(x+2)^2=24$$
$$x+2=\pm\sqrt{24}$$
$$x+2=\pm 2\sqrt{6}$$
$$x=-2\pm 2\sqrt{6}$$

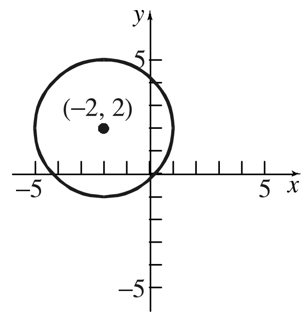
y-intercepts: $(0+2)^2+(y+1)^2=5^2$
$$4+(y+1)^2=25$$
$$(y+1)^2=21$$
$$y+1=\pm\sqrt{21}$$
$$y=-1\pm\sqrt{21}$$

The intercepts are $\left(-2-2\sqrt{6},0\right)$, $\left(-2+2\sqrt{6},0\right)$, $\left(0,-1-\sqrt{21}\right)$, and $\left(0,-1+\sqrt{21}\right)$.

27.
$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

b.



c. x-intercepts: $(x+2)^2+(0-2)^2=3^2$
$$(x+2)^2+4=9$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

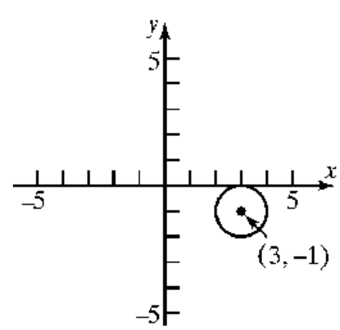
y-intercepts: $(0+2)^2+(y-2)^2=3^2$
$$4+(y-2)^2=9$$
$$(y-2)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

The intercepts are $\left(-2-\sqrt{5},0\right)$, $\left(-2+\sqrt{5},0\right)$, $\left(0,2-\sqrt{5}\right)$, and $\left(0,2+\sqrt{5}\right)$.

28.
$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts: $(x-3)^2+(0+1)^2=1^2$

$$(x-3)^2+1=1$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

y-intercepts: $(0-3)^2+(y+1)^2=1^2$

$$9+(y+1)^2=1$$
$$(y+1)^2=-8$$

No real solution.

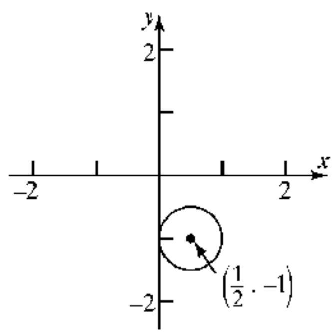
The intercept only intercept is $(3,0)$.

29.

$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius = $\frac{1}{2}$

b.



c. x-intercepts: $\left(x-\frac{1}{2}\right)^2+(0+1)^2=\left(\frac{1}{2}\right)^2$

$$\left(x-\frac{1}{2}\right)^2+1=\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2=-\frac{3}{4}$$

No real solutions

y-intercepts: $\left(0-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$

$$\frac{1}{4}+(y+1)^2=\frac{1}{4}$$
$$(y+1)^2=0$$
$$y+1=0$$
$$y=-1$$

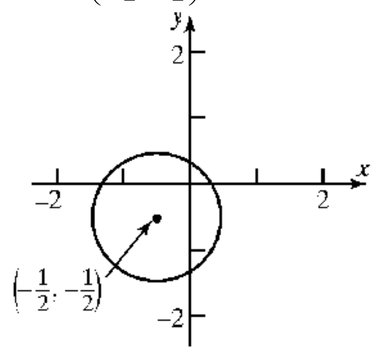
The only intercept is $(0,-1)$.

30.

$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=1^2$

$$\left(x+\frac{1}{2}\right)^2+\frac{1}{4}=1$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$x=\frac{-1\pm\sqrt{3}}{2}$$

y-intercepts: $\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$

$$\frac{1}{4}+\left(y+\frac{1}{2}\right)^2=1$$
$$\left(y+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$y+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$y=\frac{-1\pm\sqrt{3}}{2}$$

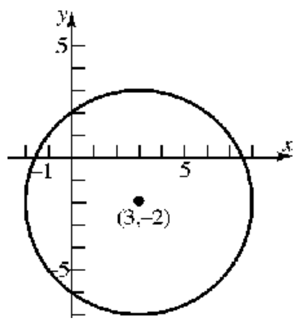
The intercepts are $\left(\frac{-1-\sqrt{3}}{2},0\right)$, $\left(\frac{-1+\sqrt{3}}{2},0\right)$, $\left(0,\frac{-1-\sqrt{3}}{2}\right)$, and $\left(0,\frac{-1+\sqrt{3}}{2}\right)$.

31. $2x^2+2y^2-12x+8y-24=0$

$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$

$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

y-intercepts: $(0-3)^2+(y+2)^2=5^2$

$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm 4$$
$$y=-2\pm 4$$
$$y=2 \quad \text{or} \quad y=-6$$

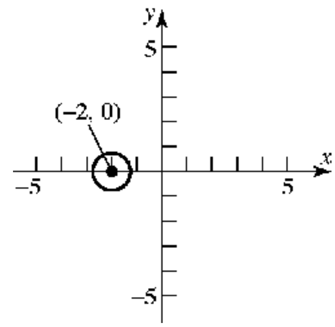
The intercepts are $\left(3-\sqrt{21},0\right)$, $\left(3+\sqrt{21},0\right)$, $(0,-6)$, and $(0,2)$.

32. a. $2x^2+2y^2+8x+7=0$

$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\frac{\sqrt{2}}{2}$

b.



c. x-intercepts: $(x+2)^2+(0)^2=\frac{1}{2}$

$$(x+2)^2=\frac{1}{2}$$
$$x+2=\pm\sqrt{\frac{1}{2}}$$
$$x+2=\pm\frac{\sqrt{2}}{2}$$
$$x=-2\pm\frac{\sqrt{2}}{2}$$

y-intercepts: $(0+2)^2+y^2=\frac{1}{2}$

$$4+y^2=\frac{1}{2}$$
$$y^2=-\frac{7}{2}$$

No real solutions.

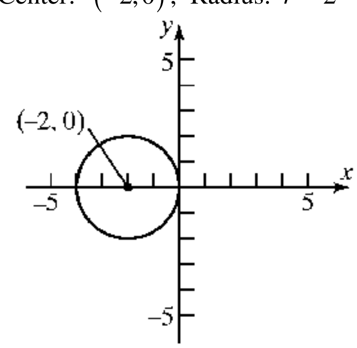
The intercepts are $\left(-2-\frac{\sqrt{2}}{2},0\right)$ and $\left(-2+\frac{\sqrt{2}}{2},0\right)$.

33. $2x^2+8x+2y^2=0$

$$x^2+4x+y^2=0$$
$$x^2+4x+4+y^2=0+4$$
$$(x+2)^2+y^2=2^2$$

a. Center: $(-2,0)$; Radius: $r=2$

b.



c. x-intercepts: $(x+2)^2+(0)^2=2^2$

$$(x+2)^2=4$$
$$(x+2)^2=\pm\sqrt{4}$$
$$x+2=\pm 2$$
$$x=-2\pm 2$$
$$x=0 \quad \text{or} \quad x=-4$$

y-intercepts: $(0+2)^2+y^2=2^2$

$$4+y^2=4$$
$$y^2=0$$
$$y=0$$

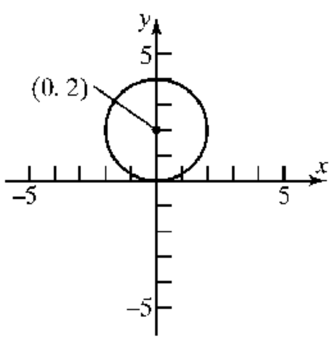
The intercepts are $(-4,0)$ and $(0,0)$.

34. $3x^2+3y^2-12y=0$

$$x^2+y^2-4y=0$$
$$x^2+y^2-4y+4=0+4$$
$$x^2+(y-2)^2=4$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts: $x^2+(0-2)^2=4$

$$x^2+4=4$$
$$x^2=0$$
$$x=0$$

y-intercepts: $0^2+(y-2)^2=4$

$$(y-2)^2=4$$
$$y-2=\pm\sqrt{4}$$
$$y-2=\pm 2$$
$$y=2\pm 2$$
$$y=4 \quad \text{or} \quad y=0$$

The intercepts are $(0,0)$ and $(0,4)$.

35. Center at (0, 0); containing point (–2, 3).

$$r=\sqrt{(-2-0)^2+(3-0)^2}=\sqrt{4+9}=\sqrt{13}$$

Equation: $(x-0)^2+(y-0)^2=\left(\sqrt{13}\right)^2$

$$x^2+y^2=13$$

36. Center at (1, 0); containing point (–3, 2).

$$r=\sqrt{(-3-1)^2+(2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

Equation: $(x-1)^2+(y-0)^2=\left(\sqrt{20}\right)^2$

$$(x-1)^2+y^2=20$$

37. Center at (2, 3); tangent to the x-axis.

$r=3$

Equation: $(x-2)^2+(y-3)^2=3^2$

$$(x-2)^2+(y-3)^2=9$$

38. Center at (–3, 1); tangent to the y-axis.

$r=3$

Equation: $(x+3)^2+(y-1)^2=3^2$

$$(x+3)^2+(y-1)^2=9$$

39. Endpoints of a diameter are (1, 4) and (–3, 2).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{1+(-3)}{2},\dfrac{4+2}{2}\right)=(-1,3)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-(-1))^2+(y-3)^2=\left(\sqrt{5}\right)^2$

$$(x+1)^2+(y-3)^2=5$$

40. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:

Center: $\left(\dfrac{4+0}{2},\dfrac{3+1}{2}\right)=(2,2)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-2)^2+(y-2)^2=\left(\sqrt{5}\right)^2$

$$(x-2)^2+(y-2)^2=5$$

41. Center at (–1, 3); tangent to the line $y = 2$. This means that the circle contains the point (–1, 2), so the radius is $r = 1$.

Equation: $(x+1)^2 + (y-3)^2 = (1)^2$

$$(x+1)^2 + (y-3)^2 = 1$$

42. Center at (4, –2); tangent to the line $x = 1$. This means that the circle contains the point (1, –2), so the radius is $r = 3$.

Equation: $(x-4)^2 + (y+2)^2 = (3)^2$

$$(x-4)^2 + (y+2)^2 = 9$$

43. (c); Center: $(1,-2)$; Radius = 2

44. (d) ; Center: $(-3,3)$; Radius = 3

45. (b) ; Center: $(-1,2)$; Radius = 2

46. (a) ; Center: $(-3,3)$; Radius = 3

47. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 9$$
$$x^2 + x^2 = 9$$
$$2x^2 = 9$$
$$x^2 = \frac{9}{2}$$
$$x = \sqrt{\frac{9}{2}} = \frac{3\sqrt{2}}{2}$$

The length of one side of the square is $2x$. Thus, the area is

$$A = s^2 = \left(2 \cdot \frac{3\sqrt{2}}{2}\right)^2 = \left(3\sqrt{2}\right)^2 = 18 \text{ square units.}$$

48. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point (x, y). The circle and the square are both centered about the origin. Because of symmetry, we have that $x = y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 36$$
$$x^2 + x^2 = 36$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2 \cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is $\pi r^2 = \pi(6)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

49. The diameter of the Ferris wheel was 250 feet, so the radius was 125 feet. The maximum height was 264 feet, so the center was at a height of $264 - 125 = 139$ feet above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 139). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-139)^2 = 125^2$$
$$x^2 + (y-139)^2 = 15,625$$

50. The diameter of the wheel is 150 meters, so the radius is 75 meters. The maximum height is 165 meters, so the center of the wheel is at a height of $167 - 75 = 90$ meters above the ground. Since the center of the wheel is on the *y*-axis, it is the point (0, 90). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-90)^2 = 75^2$$
$$x^2 + (y-90)^2 = 5625$$

51.
$$x^2 + y^2 + 2x + 4y - 4091 = 0$$
$$x^2 + 2x + y^2 + 4y - 4091 = 0$$
$$x^2 + 2x + 1 + y^2 + 4y + 4 = 4091 + 5$$
$$(x+1)^2 + (y+2)^2 = 4096$$

The circle representing Earth has center $(-1,-2)$ and radius = $\sqrt{4096} = 64$.
So the radius of the satellite's orbit is $64 + 0.6 = 64.6$ units.
The equation of the orbit is

$$(x+1)^2 + (y+2)^2 = (64.6)^2$$
$$x^2 + y^2 + 2x + 4y - 4168.16 = 0$$

52. **a.**

$$x^2+(mx+b)^2=r^2$$
$$x^2+m^2x^2+2bmx+b^2=r^2$$
$$(1+m^2)x^2+2bmx+b^2-r^2=0$$

There is one solution if and only if the discriminant is zero.

$$(2bm)^2-4(1+m^2)(b^2-r^2)=0$$
$$4b^2m^2-4b^2+4r^2-4b^2m^2+4m^2r^2=0$$
$$-4b^2+4r^2+4m^2r^2=0$$
$$-b^2+r^2+m^2r^2=0$$
$$r^2(1+m^2)=b^2$$

b. Using the quadratic formula, the result from part (a), and knowing that the discriminant is zero, we get:

$$(1+m^2)x^2+2bmx+b^2-r^2=0$$
$$x=\frac{-2bm}{2(1+m^2)}=\frac{-bm}{\left(\frac{b^2}{r^2}\right)}=\frac{-bmr^2}{b^2}=\frac{-mr^2}{b}$$
$$y=m\left(\frac{-mr^2}{b}\right)+b$$
$$=\frac{-m^2r^2}{b}+b=\frac{-m^2r^2+b^2}{b}=\frac{r^2}{b}$$

c. The slope of the tangent line is m.
The slope of the line joining the point of tangency and the center is:

$$\frac{\left(\frac{r^2}{b}-0\right)}{\left(\frac{-mr^2}{b}-0\right)}=\frac{r^2}{b}\cdot\frac{b}{-mr^2}=-\frac{1}{m}$$

Therefore, the tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

53. $x^2+y^2=9$

Center: (0, 0)

Slope from center to $\left(1,2\sqrt{2}\right)$ is

$$\frac{2\sqrt{2}-0}{1-0}=\frac{2\sqrt{2}}{1}=2\sqrt{2}.$$

Slope of the tangent line is $\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$.

Equation of the tangent line is:

$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}(x-1)$$
$$y-2\sqrt{2}=-\frac{\sqrt{2}}{4}x+\frac{\sqrt{2}}{4}$$
$$4y-8\sqrt{2}=-\sqrt{2}\,x+\sqrt{2}$$
$$\sqrt{2}\,x+4y=9\sqrt{2}$$
$$\sqrt{2}\,x+4y-9\sqrt{2}=0$$

54. $x^2+y^2-4x+6y+4=0$

$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: (2, –3)

Slope from center to $\left(3,2\sqrt{2}-3\right)$ is

$$\frac{2\sqrt{2}-3-(-3)}{3-2}=\frac{2\sqrt{2}}{1}=2\sqrt{2}$$

Slope of the tangent line is: $\frac{-1}{2\sqrt{2}}=-\frac{\sqrt{2}}{4}$

Equation of the tangent line:

$$y-\left(2\sqrt{2}-3\right)=-\frac{\sqrt{2}}{4}(x-3)$$
$$y-2\sqrt{2}+3=-\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}$$
$$4y-8\sqrt{2}+12=-\sqrt{2}x+3\sqrt{2}$$
$$\sqrt{2}x+4y-11\sqrt{2}+12=0$$

55. Let (h,k) be the center of the circle.

$$x-2y+4=0$$
$$2y=x+4$$
$$y=\frac{1}{2}x+2$$

The slope of the tangent line is $\frac{1}{2}$. The slope from (h,k) to (0, 2) is –2.

$$\frac{2-k}{0-h}=-2$$
$$2-k=2h$$

The other tangent line is $y=2x-7$, and it has slope 2.

The slope from (h, k) to $(3, -1)$ is $-\frac{1}{2}$.

$$\frac{-1-k}{3-h} = -\frac{1}{2}$$
$$2+2k = 3-h$$
$$2k = 1-h$$
$$h = 1-2k$$

Solve the two equations in h and k:

$$2-k = 2(1-2k)$$
$$2-k = 2-4k$$
$$3k = 0$$
$$k = 0$$
$$h = 1-2(0) = 1$$

The center of the circle is (1, 0).

56. Find the centers of the two circles:

$$x^2 + y^2 - 4x + 6y + 4 = 0$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 9$$

Center: $(2, -3)$

$$x^2 + y^2 + 6x + 4y + 9 = 0$$
$$(x^2 + 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$
$$(x+3)^2 + (y+2)^2 = 4$$

Center: $(-3, -2)$

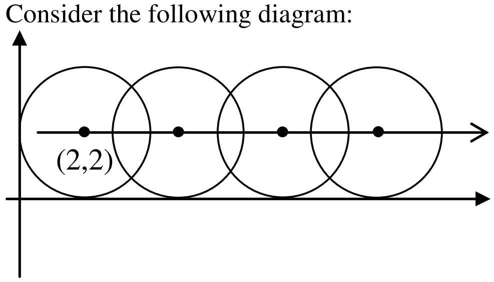
Find the slope of the line containing the centers:

$$m = \frac{-2-(-3)}{-3-2} = -\frac{1}{5}$$

Find the equation of the line containing the centers:

$$y + 3 = -\frac{1}{5}(x-2)$$
$$5y + 15 = -x + 2$$
$$x + 5y = -13$$
$$x + 5y + 13 = 0$$

57. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y = 2$.

58.
$$C = 2\pi r$$
$$6\pi = 2\pi r$$
$$\frac{6\pi}{2\pi} = \frac{2\pi r}{2\pi}$$
$$3 = r$$

The radius is 3 units long.

59. (b), (c), (e) and (g)
We need $h, k > 0$ and $(0, 0)$ on the graph.

60. (b), (e) and (g)
We need $h < 0$, $k = 0$, and $|h| > r$.

61. Answers will vary.

62. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x-h)^2 + (y-k)^2 = r^2$.

$$(x+3)^2 + (y-2)^2 = 16$$
$$\left(x-(-3)\right)^2 + (y-2)^2 = 4^2$$

Thus, $(h, k) = (-3, 2)$ and $r = 4$.

63 – 64. Interactive Exercises.

Section 2.5

1. $y = kx$

2. False. If y varies directly with x, then $y = kx$, where k is a constant.

3.
$$y = kx$$
$$2 = 10k$$
$$k = \frac{2}{10} = \frac{1}{5}$$
$$y = \frac{1}{5}x$$

4.
$$v = kt$$
$$16 = 2k$$
$$8 = k$$
$$v = 8t$$

5. $A = kx^2$

$4\pi = k(2)^2$

$4\pi = 4k$

$\pi = k$

$A = \pi x^2$

6. $V = kx^3$

$36\pi = k(3)^3$

$36\pi = 27k$

$k = \frac{36\pi}{27} = \frac{4}{3}\pi$

$V = \frac{4}{3}\pi x^3$

7. $F = \frac{k}{d^2}$

$10 = \frac{k}{5^2}$

$10 = \frac{k}{25}$

$k = 250$

$F = \frac{250}{d^2}$

8. $y = \frac{k}{\sqrt{x}}$

$4 = \frac{k}{\sqrt{9}}$

$4 = \frac{k}{3}$

$k = 12$

$y = \frac{12}{\sqrt{x}}$

9. $z = k\left(x^2 + y^2\right)$

$5 = k\left(3^2 + 4^2\right)$

$5 = k(25)$

$k = \frac{5}{25} = \frac{1}{5}$

$z = \frac{1}{5}\left(x^2 + y^2\right)$

10. $T = k\left(\sqrt[3]{x}\right)\left(d^2\right)$

$18 = k\left(\sqrt[3]{8}\right)\left(3^2\right)$

$18 = k\left(18\right)$

$1 = k$

$T = \left(\sqrt[3]{x}\right)\left(d^2\right)$

11. $M = \frac{kd^2}{\sqrt{x}}$

$24 = \frac{k\left(4^2\right)}{\sqrt{9}}$

$24 = \frac{16k}{3}$

$k = 24\left(\frac{3}{16}\right) = \frac{9}{2}$

$M = \frac{9d^2}{2\sqrt{x}}$

12. $z = k\left(x^3 + y^2\right)$

$1 = k\left(2^3 + 3^2\right)$

$1 = k\left(17\right)$

$k = \frac{1}{17}$

$z = \frac{1}{17}\left(x^3 + y^2\right)$

13. $T^2 = \frac{ka^3}{d^2}$

$2^2 = \frac{k\left(2^3\right)}{4^2}$

$4 = \frac{k\left(8\right)}{16}$

$4 = \frac{k}{2}$

$k = 8$

$T^2 = \frac{8a^3}{d^2}$

14. $z^3 = k\left(x^2 + y^2\right)$

$2^3 = k\left(9^2 + 4^2\right)$

$8 = k(97)$

$k = \frac{8}{97}$

$z^3 = \frac{8}{97}\left(x^2 + y^2\right)$

15. $V = \frac{4\pi}{3} r^3$

16. $c^2 = a^2 + b^2$

17. $A = \frac{1}{2} bh$

18. $p = 2(l + w)$

19. $F = \left(6.67 \times 10^{-11}\right)\left(\frac{mM}{d^2}\right)$

20. $T = \frac{2\pi}{\sqrt{32}}\sqrt{l}$

21. $p = kB$

$6.49 = k(1000)$

$0.00649 = k$

Therefore we have the linear equation $p = 0.00649B$.

If $B = 145000$, then

$p = 0.00649(145000) = \$941.05$.

22. $p = kB$

$8.99 = k(1000)$

$0.00899 = k$

Therefore we have the linear equation $p = 0.00899B$.

If $B = 175000$, then

$p = 0.00899(175000) = \$1573.25$.

23. $s = kt^2$

$16 = k(1)^2$

$k = 16$

Therefore, we have equation $s = 16t^2$.

If $t = 3$ seconds, then $s = 16(3)^2 = 144$ feet.

If $s = 64$ feet, then

$64 = 16t^2$

$t^2 = 4$

$t = \pm 2$

Time must be positive, so we disregard $t = -2$.

It takes 2 seconds to fall 64 feet.

24. $v = kt$

$64 = k(2)$

$k = 32$

Therefore, we have the linear equation $v = 32t$.

If $t = 3$ seconds, then $v = 32(3) = 96$ ft/sec.

25. $E = kW$

$3 = k(20)$

$k = \frac{3}{20}$

Therefore, we have the linear equation $E = \frac{3}{20}W$.

If $W = 15$, then $E = \frac{3}{20}(15) = 2.25$.

26. $R = \frac{k}{l}$

$256 = \frac{k}{48}$

$k = 12,288$

Therefore, we have the equation $R = \frac{12,288}{l}$.

If $R = 576$, then

$576 = \frac{12,288}{l}$

$576l = 12,288$

$l = \frac{12,288}{576} = \frac{64}{3}$ inches

27. $R = kg$

$47.40 = k(12)$

$3.95 = k$

Therefore, we have the linear equation $R = 3.95g$.

If $g = 10.5$, then $R = (3.95)(10.5) \approx \$41.48$.

28. $C = kA$

$23.75 = k(5)$

$4.75 = k$

Therefore, we have the linear equation $C = 4.75A$.

If $A = 3.5$, then $C = (4.75)(3.5) = \$16.63$.

29. $D = \dfrac{k}{p}$

a. $D = 156$, $p = 2.75$;

$156 = \dfrac{k}{2.75}$

$k = 429$

So, $D = \dfrac{429}{p}$.

b. $D = \dfrac{429}{3} = 143$ bags of candy

30. $t = \dfrac{k}{s}$

a. $t = 40$, $s = 30$;

$40 = \dfrac{k}{30}$

$k = 1200$

So, we have the equation $t = \dfrac{1200}{s}$.

b. $t = \dfrac{1200}{40} = 30$ minutes

31. $V = \dfrac{k}{P}$

$V = 600, P = 150$;

$600 = \dfrac{k}{150}$

$k = 90{,}000$

So, we have the equation $V = \dfrac{90{,}000}{P}$

If $P = 200$, then $V = \dfrac{90{,}000}{200} = 450 \text{ cm}^3$.

32. $i = \dfrac{k}{R}$

If $i = 30, R = 8$, then $30 = \dfrac{k}{8}$ and $k = 240$.

So, we have the equation $i = \dfrac{240}{R}$.

If $R = 10$, then $i = \dfrac{240}{10} = 24$ amperes.

33. $W = \dfrac{k}{d^2}$

If $W = 125, d = 3960$ then

$125 = \dfrac{k}{3960^2}$ and $k = 1{,}960{,}200{,}000$

So, we have the equation $W = \dfrac{1{,}960{,}200{,}000}{d}$.

At the top of Mt. McKinley, we have

$d = 3960 + 3.8 = 3963.8$, so

$W = \dfrac{1{,}960{,}200{,}000}{(3963.8)^2} \approx 124.76$ pounds.

34. $I = \dfrac{k}{d^2}$

If $I = 0.075, d = 2$, then

$0.075 = \dfrac{k}{2^2}$ and $k = 0.3$.

So, we have the equation $I = \dfrac{0.3}{d^2}$.

If $d = 5$, then $I = \dfrac{0.3}{5^2} = 0.012$ foot-candles.

35. $V = \pi r^2 h$

36. $V = \dfrac{\pi}{3} r^2 h$

37. $W = \dfrac{k}{d^2}$

$55 = \dfrac{k}{3960^2}$

$k = 862{,}488{,}000$

So, we have the equation $W = \dfrac{862{,}488{,}000}{d^2}$.

If d=3965, then

$W = \dfrac{862{,}488{,}000}{3965^2} \approx 54.86$ pounds.

38. $F = kAv^2$

$11 = k(20)(22)^2$

$11 = 9860k$

$k = \frac{11}{9680} = \frac{1}{880}$

So, we have the equation $F = \frac{1}{880}Av^2$.

If $A = 47.125$ and $v = 36.5$, then

$F = \frac{1}{880}(47.125)(36.5)^2 \approx 71.34$ pounds.

39. $h = ksd^3$

$36 = k(75)(2)^3$

$36 = 600k$

$0.06 = k$

So, we have the equation $h = 0.06sd^3$.
If $h = 45$ and $s = 125$, then

$45 = (0.06)(125)d^3$

$45 = 7.5d^3$

$6 = d^3$

$d = \sqrt[3]{6} \approx 1.82$ inches

40. $V = \frac{kT}{P}$

$100 = \frac{k(300)}{15}$

$100 = 20k$

$5 = k$

So, we have the equation $V = \frac{5T}{P}$.

If $V = 80$ and $T = 310$, then

$80 = \frac{5(310)}{P}$

$80P = 1550$

$P = \frac{1550}{80} = 19.375$ atmospheres

41. $K = kmv^2$

$1250 = k(25)(10)^2$

$1250 = 2500k$

$k = 0.5$

So, we have the equation $K = 0.5mv^2$.
If $m = 25$ and $v = 15$, then

$K = 0.5(25)(15)^2 = 2812.5$ Joules

42. $R = \frac{kl}{d^2}$

$1.24 = \frac{k(432)}{(4)^2}$

$1.24 = 27k$

$k = \frac{1.24}{27}$

So, we have the equation $R = \frac{1.24l}{27d^2}$.

If R = 1.44 and d = 3, then

$1.44 = \frac{1.24l}{27(3)^2}$

$1.44 = \frac{1.24l}{243}$

$349.92 = 1.24l$

$l = \frac{349.92}{1.24} \approx 282.2$ feet

43. $S = \frac{kpd}{t}$

$100 = \frac{k(25)(5)}{0.75}$

$75 = 125k$

$0.6 = k$

So, we have the equation $S = \frac{0.6pd}{t}$.

If $p = 40$, $d = 8$, and $t = 0.50$, then

$S = \frac{0.6(40)(8)}{0.50} = 384$ psi.

44. $S = \frac{kwt^2}{l}$

$750 = \frac{k(4)(2)^2}{8}$

$750 = 2k$

$375 = k$

So, we have the equation $S = \frac{375wt^2}{l}$.

If $l = 10$, $w = 6$, and $t = 2$, then

$$S = \frac{375(6)(2)^2}{10} = 900 \text{ pounds.}$$

45 – 48. Answers will vary.

Chapter 2 Review Exercises

1. $P_1 = (0,0)$ and $P_2 = (4,2)$

a. $d(P_1, P_2) = \sqrt{(4-0)^2 + (2-0)^2}$
$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{0+4}{2}, \frac{0+2}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

d. For each run of 2, there is a rise of 1.

2. $P_1 = (0,0)$ and $P_2 = (-4,6)$

a. $d(P_1, P_2) = \sqrt{(-4-0)^2 + (6-0)^2}$
$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{0+(-4)}{2}, \frac{0+6}{2}\right) = \left(\frac{-4}{2}, \frac{6}{2}\right) = (-2,3)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{6-0}{-4-0} = \frac{6}{-4} = -\frac{3}{2}$

d. For each run of 2, there is a rise of -3.

3. $P_1 = (1,-1)$ and $P_2 = (-2,3)$

a. $d(P_1, P_2) = \sqrt{(-2-1)^2 + (3-(-1))^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{1+(-2)}{2}, \frac{-1+3}{2}\right)$$
$$= \left(\frac{-1}{2}, \frac{2}{2}\right) = \left(-\frac{1}{2}, 1\right)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3-(-1)}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$

d. For each run of 3, there is a rise of -4.

4. $P_1 = (-2,2)$ and $P_2 = (1,4)$

a. $d(P_1, P_2) = \sqrt{(1-(-2))^2 + (4-2)^2}$
$= \sqrt{9+4} = \sqrt{13}$

b. $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{-2+1}{2}, \frac{2+4}{2}\right)$$
$$= \left(-\frac{1}{2}, \frac{6}{2}\right) = \left(-\frac{1}{2}, 3\right)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{4-2}{1-(-2)} = \frac{2}{3}$

d. For each run of 3, there is a rise of 2.

5. $P_1 = (4,-4)$ and $P_2 = (4,8)$

a. $d(P_1, P_2) = \sqrt{(4-4)^2 + (8-(-4))^2}$
$= \sqrt{0+144} = \sqrt{144} = 12$

b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{4+4}{2}, \frac{-4+8}{2}\right) = \left(\frac{8}{2}, \frac{4}{2}\right) = (4,2)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-(-4)}{4-4} = \frac{12}{0}$, undefined

d. An undefined slope means the points lie on a vertical line. There is no change in x.

6. $P_1 = (-3,4)$ and $P_2 = (2,4)$

a. $d(P_1,P_2) = \sqrt{(2-(-3))^2 + (4-4)^2}$
$= \sqrt{25+0} = \sqrt{25} = 5$

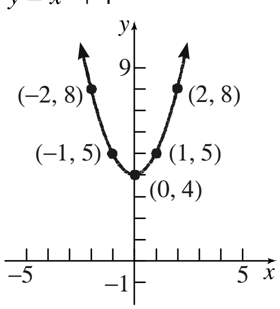
b. The coordinates of the midpoint are:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-3+2}{2}, \frac{4+4}{2}\right) = \left(-\frac{1}{2}, \frac{8}{2}\right) = \left(-\frac{1}{2}, 4\right)$$

c. $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{4-4}{2-(-3)} = \frac{0}{5} = 0$

d. A slope of zero means the points lie on a horizontal line. There is no change in *y*.

7. $y = x^2 + 4$



8. *x*-intercepts: $-4, 0, 2$; *y*-intercepts: $-2, 0, 2$
Intercepts: $(-4, 0), (0, 0), (2, 0), (0, -2), (0, 2)$

9. $2x = 3y^2$

x-intercepts:
$2x = 3(0)^2$
$2x = 0$
$x = 0$

y-intercepts:
$2(0) = 3y^2$
$0 = y^2$
$y = 0$

The only intercept is $(0, 0)$.

<u>Test *x*-axis symmetry</u>: Let $y = -y$
$2x = 3(-y)^2$
$2x = 3y^2$ same

<u>Test *y*-axis symmetry</u>: Let $x = -x$
$2(-x) = 3y^2$
$-2x = 3y^2$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$2(-x) = 3(-y)^2$
$-2x = 3y^2$ different

Therefore, the graph will have *x*-axis symmetry.

10. $y = 5x$

x-intercepts:
$0 = 5x$
$0 = x$

y-intercepts:
$y = 5(0)$
$y = 0$

The only intercept is $(0, 0)$.

<u>Test *x*-axis symmetry</u>: Let $y = -y$
$-y = 5x$
$y = -5x$ different

<u>Test *y*-axis symmetry</u>: Let $x = -x$
$y = 5(-x)$
$y = -5x$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$-y = 5(-x)$
$y = 5x$ same

Therefore, the graph will have origin symmetry.

11. $x^2 + 4y^2 = 16$

x-intercepts:
$x^2 + 4(0)^2 = 16$
$x^2 = 16$
$x = \pm 4$

y-intercepts:
$(0)^2 + 4y^2 = 16$
$4y^2 = 16$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4, 0), (4, 0), (0, -2)$, and $(0, 2)$.

<u>Test *x*-axis symmetry</u>: Let $y = -y$
$x^2 + 4(-y)^2 = 16$
$x^2 + 4y^2 = 16$ same

<u>Test *y*-axis symmetry</u>: Let $x = -x$
$(-x)^2 + 4y^2 = 16$
$x^2 + 4y^2 = 16$ same

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.
$(-x)^2 + 4(-y)^2 = 16$
$x^2 + 4y^2 = 16$ same

Therefore, the graph will have *x*-axis, *y*-axis, and origin symmetry.

12. $9x^2 - y^2 = 9$

x-intercepts:

$$9x^2 - (0)^2 = 9$$
$$9x^2 = 9$$
$$x^2 = 1$$
$$x = \pm 1$$

y-intercepts:

$$9(0)^2 - y^2 = 9$$
$$-y^2 = 9$$
$$y^2 = -9$$

no real solutions

The intercepts are $(-1, 0)$ and $(1, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$$9x^2 - (-y)^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

Test *y*-axis symmetry: Let $x = -x$

$$9(-x)^2 - y^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$9(-x)^2 - (-y)^2 = 9$$
$$9x^2 - y^2 = 9 \quad \text{same}$$

Therefore, the graph will have *x*-axis, *y*-axis, and origin symmetry.

13. $y = x^4 + 2x^2 + 1$

x-intercepts:

$$0 = x^4 + 2x^2 + 1$$
$$0 = (x^2 + 1)(x^2 + 1)$$
$$x^2 + 1 = 0$$
$$x^2 = -1$$

no real solutions

y-intercepts:

$$y = (0)^4 + 2(0)^2 + 1$$
$$= 1$$

The only intercept is (0, 1).

Test *x*-axis symmetry: Let $y = -y$

$$-y = x^4 + 2x^2 + 1$$
$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Test *y*-axis symmetry: Let $x = -x$

$$y = (-x)^4 + 2(-x)^2 + 1$$
$$y = x^4 + 2x^2 + 1 \quad \text{same}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^4 + 2(-x)^2 + 1$$
$$-y = x^4 + 2x^2 + 1$$
$$y = -x^4 - 2x^2 - 1 \quad \text{different}$$

Therefore, the graph will have *y*-axis symmetry.

14. $y = x^3 - x$

x-intercepts:

$$0 = x^3 - x$$
$$0 = x(x^2 - 1)$$
$$0 = x(x+1)(x-1)$$
$$x = 0,\ x = -1,\ x = 1$$

y-intercepts:

$$y = (0)^3 - 0$$
$$= 0$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(1, 0)$.

Test *x*-axis symmetry: Let $y = -y$

$$-y = x^3 - x$$
$$y = -x^3 + x \quad \text{different}$$

Test *y*-axis symmetry: Let $x = -x$

$$y = (-x)^3 - (-x)$$
$$y = -x^3 + x \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$-y = (-x)^3 - (-x)$$
$$-y = -x^3 + x$$
$$y = x^3 - x \quad \text{same}$$

Therefore, the graph will have origin symmetry.

15. $x^2 + x + y^2 + 2y = 0$

x-intercepts:

$$x^2 + x + (0)^2 + 2(0) = 0$$
$$x^2 + x = 0$$
$$x(x+1) = 0$$
$$x = 0,\ x = -1$$

y-intercepts:

$$(0)^2 + 0 + y^2 + 2y = 0$$
$$y^2 + 2y = 0$$
$$y(y+2) = 0$$
$$y = 0,\ y = -2$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test *x*-axis symmetry: Let $y = -y$

$$x^2 + x + (-y)^2 + 2(-y) = 0$$
$$x^2 + x + y^2 - 2y = 0 \quad \text{different}$$

Test *y*-axis symmetry: Let $x = -x$

$$(-x)^2 + (-x) + y^2 + 2y = 0$$
$$x^2 - x + y^2 + 2y = 0 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.

$$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$$
$$x^2 - x + y^2 - 2y = 0 \quad \text{different}$$

The graph has none of the indicated symmetries.

16. $x^2+4x+y^2-2y=0$

x-intercepts: $x^2+4x+(0)^2-2(0)=0$
$x^2+4x=0$
$x(x+4)=0$
$x=0,\ x=-4$

y-intercepts: $(0)^2+4(0)+y^2-2y=0$
$y^2-2y=0$
$y(y-2)=0$
$y=0,\ y=2$

The intercepts are $(-4, 0)$, $(0, 0)$, and $(0, 2)$.

Test x-axis symmetry: Let $y=-y$

$x^2+4x+(-y)^2-2(-y)=0$
$x^2+4x+y^2+2y=0$ different

Test y-axis symmetry: Let $x=-x$

$(-x)^2+4(-x)+y^2-2y=0$
$x^2-4x+y^2-2y=0$ different

Test origin symmetry: Let $x=-x$ and $y=-y$.

$(-x)^2+4(-x)+(-y)^2-2(-y)=0$
$x^2-4x+y^2+2y=0$ different

The graph has none of the indicated symmetries.

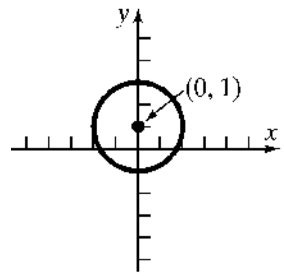
17. $(x-h)^2+(y-k)^2=r^2$
$(x-(-2))^2+(y-3)^2=4^2$
$(x+2)^2+(y-3)^2=16$

18. $(x-h)^2+(y-k)^2=r^2$
$(x-3)^2+(y-4)^2=4^2$
$(x-3)^2+(y-4)^2=16$

19. $(x-h)^2+(y-k)^2=r^2$
$(x-(-1))^2+(y-(-2))^2=1^2$
$(x+1)^2+(y+2)^2=1$

20. $(x-h)^2+(y-k)^2=r^2$
$(x-2)^2+(y-(-4))^2=3^2$
$(x-2)^2+(y+4)^2=9$

21. $x^2+(y-1)^2=4$
$x^2+(y-1)^2=2^2$
Center: (0,1); Radius = 2

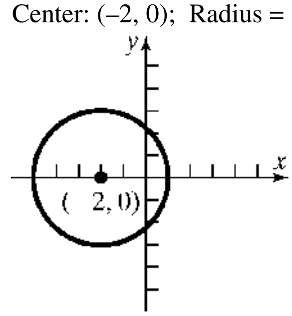


x-intercepts: $x^2+(0-1)^2=4$
$x^2+1=4$
$x^2=3$
$x=\pm\sqrt{3}$

y-intercepts: $0^2+(y-1)^2=4$
$(y-1)^2=4$
$y-1=\pm 2$
$y=1\pm 2$
$y=3$ or $y=-1$

The intercepts are $(-\sqrt{3}, 0)$, $(\sqrt{3}, 0)$, $(0,-1)$, and $(0, 3)$.

22. $(x+2)^2+y^2=9$
$(x+2)^2+y^2=3^2$
Center: (–2, 0); Radius = 3



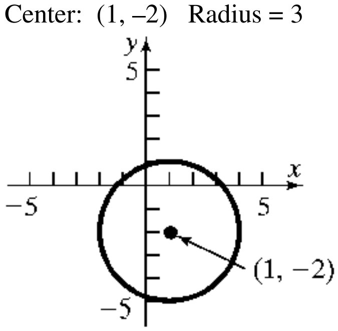
x-intercepts: $(x+2)^2+0^2=9$
$(x+2)^2=9$
$x+2=\pm 3$
$x=-2\pm 3$
$x=1$ or $x=-5$

y-intercepts:
$$\begin{aligned}(0+2)^2+y^2&=9\\4+y^2&=9\\y^2&=5\\y&=\pm\sqrt{5}\end{aligned}$$

The intercepts are $(-5, 0)$, $(1, 0)$, $(0, -\sqrt{5})$, and $(0, \sqrt{5})$.

23.
$$\begin{aligned}x^2+y^2-2x+4y-4&=0\\x^2-2x+y^2+4y&=4\\(x^2-2x+1)+(y^2+4y+4)&=4+1+4\\(x-1)^2+(y+2)^2&=3^2\end{aligned}$$

Center: (1, –2) Radius = 3



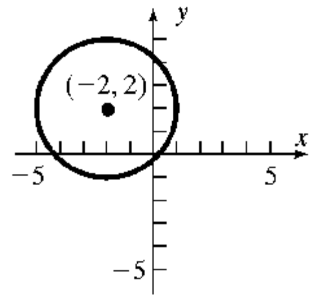
x-intercepts:
$$\begin{aligned}(x-1)^2+(0+2)^2&=3^2\\(x-1)^2+4&=9\\(x-1)^2&=5\\x-1&=\pm\sqrt{5}\\x&=1\pm\sqrt{5}\end{aligned}$$

y-intercepts:
$$\begin{aligned}(0-1)^2+(y+2)^2&=3^2\\1+(y+2)^2&=9\\(y+2)^2&=8\\y+2&=\pm\sqrt{8}\\y+2&=\pm2\sqrt{2}\\y&=-2\pm2\sqrt{2}\end{aligned}$$

The intercepts are $(1-\sqrt{5}, 0)$, $(1+\sqrt{5}, 0)$, $(0, -2-2\sqrt{2})$, and $(0, -2+2\sqrt{2})$.

24.
$$\begin{aligned}x^2+y^2+4x-4y-1&=0\\x^2+4x+y^2-4y&=1\\(x^2+4x+4)+(y^2-4y+4)&=1+4+4\\(x+2)^2+(y-2)^2&=3^2\end{aligned}$$

Center: (–2, 2) Radius = 3



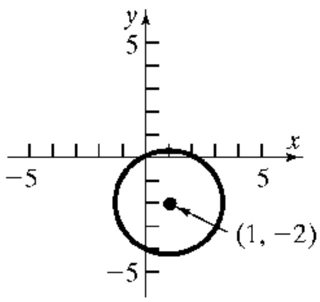
x-intercepts:
$$\begin{aligned}(x+2)^2+(0-2)^2&=3^2\\(x+2)^2+4&=9\\(x+2)^2&=5\\x+2&=\pm\sqrt{5}\\x&=-2\pm\sqrt{5}\end{aligned}$$

y-intercepts:
$$\begin{aligned}(0+2)^2+(y-2)^2&=3^2\\4+(y-2)^2&=9\\(y-2)^2&=5\\y-2&=\pm\sqrt{5}\\y&=2\pm\sqrt{5}\end{aligned}$$

The intercepts are $(-2-\sqrt{5}, 0)$, $(-2+\sqrt{5}, 0)$, $(0, 2-\sqrt{5})$, and $(0, 2+\sqrt{5})$.

25.
$$3x^2+3y^2-6x+12y=0$$
$$x^2+y^2-2x+4y=0$$
$$x^2-2x+y^2+4y=0$$
$$\left(x^2-2x+1\right)+\left(y^2+4y+4\right)=1+4$$
$$(x-1)^2+(y+2)^2=\left(\sqrt{5}\right)^2$$

Center: (1, –2) Radius = $\sqrt{5}$



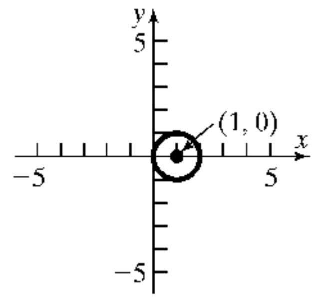
x-intercepts: $(x-1)^2+(0+2)^2=\left(\sqrt{5}\right)^2$
$$(x-1)^2+4=5$$
$$(x-1)^2=1$$
$$x-1=\pm 1$$
$$x=1\pm 1$$
$$x=2 \quad \text{or} \quad x=0$$

y-intercepts: $(0-1)^2+(y+2)^2=\left(\sqrt{5}\right)^2$
$$1+(y+2)^2=5$$
$$(y+2)^2=4$$
$$y+2=\pm 2$$
$$y=-2\pm 2$$
$$y=0 \quad \text{or} \quad y=-4$$

The intercepts are $(0,0)$, $(2,0)$, and $(0,-4)$.

26.
$$2x^2+2y^2-4x=0$$
$$x^2+y^2-2x=0$$
$$x^2-2x+y^2=0$$
$$\left(x^2-2x+1\right)+y^2=1$$
$$(x-1)^2+y^2=1^2$$

Center: (1, 0) Radius = 1



x-intercepts: $(x-1)^2+0^2=1^2$
$$(x-1)^2=1$$
$$x-1=\pm 1$$
$$x=1\pm 1$$
$$x=2 \quad \text{or} \quad x=0$$

y-intercepts: $(0-1)^2+y^2=y^2$
$$1+y^2=1$$
$$y^2=0$$
$$y=0$$

The intercepts are $(0,0)$, and $(2,0)$.

27. Slope = –2; containing (3,–1)
$$y-y_1=m(x-x_1)$$
$$y-(-1)=-2(x-3)$$
$$y+1=-2x+6$$
$$y=-2x+5 \quad \text{or} \quad 2x+y=5$$

28. Slope = 0; containing the point (–5, 4)
$$y-y_1=m(x-x_1)$$
$$y-4=0(x-(-5))$$
$$y-4=0$$
$$y=4$$

29. vertical; containing (–3,4)
Vertical lines have equations of the form $x=a$, where a is the x-intercept. Now, a vertical line containing the point (–3, 4) must have an x-intercept of –3, so the equation of the line is $x=-3$. The equation does not have a slope-intercept form.

30. x-intercept = 2; containing the point (4, –5)
Points are (2, 0) and (4, –5).

$$m = \frac{-5-0}{4-2} = -\frac{5}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{5}{2}(x-2)$$
$$y = -\frac{5}{2}x + 5 \quad \text{or} \quad 5x + 2y = 10$$

31. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)

$$m = \frac{-2-(-3)}{0-5} = \frac{1}{-5} = -\frac{1}{5}$$
$$y = mx + b$$
$$y = -\frac{1}{5}x - 2 \quad \text{or} \quad x + 5y = -10$$

32. Containing the points (3,–4) and (2, 1)

$$m = \frac{1-(-4)}{2-3} = \frac{5}{-1} = -5$$
$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -5(x-3)$$
$$y + 4 = -5x + 15$$
$$y = -5x + 11 \quad \text{or} \quad 5x + y = 11$$

33. Parallel to $2x - 3y = -4$

$$2x - 3y = -4$$
$$-3y = -2x - 4$$
$$\frac{-3y}{-3} = \frac{-2x-4}{-3}$$
$$y = \frac{2}{3}x + \frac{4}{3}$$

Slope $= \frac{2}{3}$; containing (–5,3)

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{2}{3}(x-(-5))$$
$$y - 3 = \frac{2}{3}(x+5)$$
$$y - 3 = \frac{2}{3}x + \frac{10}{3}$$
$$y = \frac{2}{3}x + \frac{19}{3} \quad \text{or} \quad 2x - 3y = -19$$

34. Parallel to $x + y = 2$

$$x + y = 2$$
$$y = -x + 2$$

Slope = –1; containing (1,–3)

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -1(x-1)$$
$$y + 3 = -x + 1$$
$$y = -x - 2 \quad \text{or} \quad x + y = -2$$

35. Perpendicular to $x + y = 2$

$$x + y = 2$$
$$y = -x + 2$$

The slope of this line is -1, so the slope of a line perpendicular to it is 1.
Slope = 1; containing (4,–3)

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 1(x-4)$$
$$y + 3 = x - 4$$
$$y = x - 7 \quad \text{or} \quad x - y = 7$$

36. Perpendicular to $3x - y = -4$

$$3x - y = -4$$
$$y = 3x + 4$$

The slope of this line is 3, so the slope of a line perpendicular to it is $-\frac{1}{3}$.

Slope $= -\frac{1}{3}$; containing (–2, 4)

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{3}(x-(-2))$$
$$y - 4 = -\frac{1}{3}x - \frac{2}{3}$$
$$y = -\frac{1}{3}x + \frac{10}{3} \quad \text{or} \quad x + 3y = 10$$

37. $4x-5y=-20$

$$-5y=-4x-20$$

$$y=\frac{4}{5}x+4$$

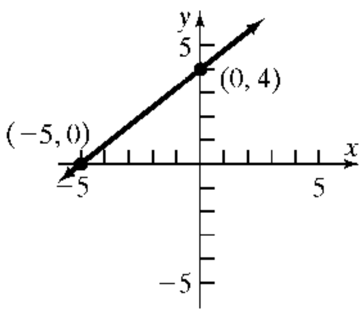
slope = $\frac{4}{5}$; y-intercept = 4

x-intercept: Let $y=0$.

$$4x-5(0)=-20$$

$$4x=-20$$

$$x=-5$$



38. $3x+4y=12$

$$4y=-3x+12$$

$$y=-\frac{3}{4}x+3$$

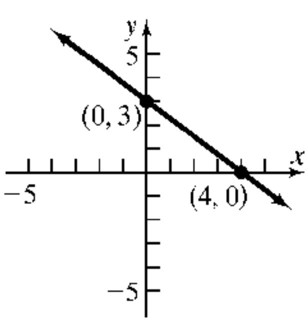
slope = $-\frac{3}{4}$; y-intercept = 3

x-intercept: Let $y=0$.

$$3x+4(0)=12$$

$$3x=12$$

$$x=4$$



39. $\frac{1}{2}x-\frac{1}{3}y=-\frac{1}{6}$

$$-\frac{1}{3}y=-\frac{1}{2}x-\frac{1}{6}$$

$$y=\frac{3}{2}x+\frac{1}{2}$$

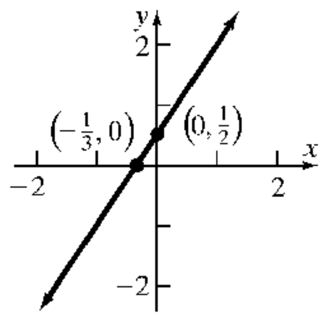
slope = $\frac{3}{2}$; y-intercept = $\frac{1}{2}$

x-intercept: Let $y=0$.

$$\frac{1}{2}x-\frac{1}{3}(0)=-\frac{1}{6}$$

$$\frac{1}{2}x=-\frac{1}{6}$$

$$x=-\frac{1}{3}$$



40. $-\frac{3}{4}x+\frac{1}{2}y=0$

$$\frac{1}{2}y=\frac{3}{4}x$$

$$y=\frac{3}{2}x$$

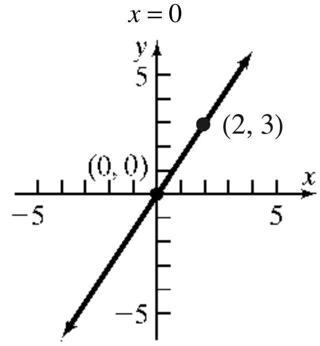
slope = $\frac{3}{2}$; y-intercept = 0

x-intercept: Let $y=0$.

$$-\frac{3}{4}x+\frac{1}{2}(0)=0$$

$$-\frac{3}{4}x=0$$

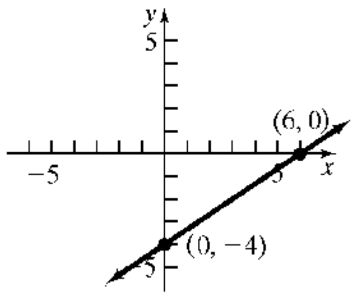
$$x=0$$



41. $2x - 3y = 12$

x-intercept:	*y*-intercept:
$2x - 3(0) = 12$	$2(0) - 3y = 12$
$2x = 12$	$-3y = 12$
$x = 6$	$y = -4$

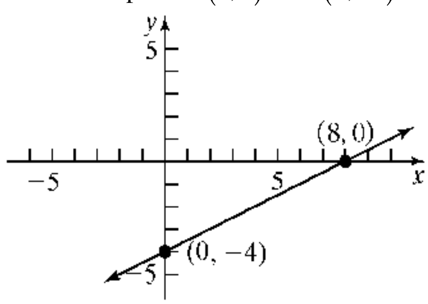
The intercepts are $(6, 0)$ and $(0, -4)$.



42. $x - 2y = 8$

x-intercept:	*y*-intercept:
$x - 2(0) = 8$	$0 - 2y = 8$
$x = 8$	$y = -4$

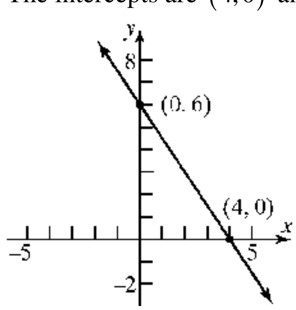
The intercepts are $(8, 0)$ and $(0, -4)$.



43. $\frac{1}{2}x + \frac{1}{3}y = 2$

x-intercept:	*y*-intercept:
$\frac{1}{2}x + \frac{1}{3}(0) = 2$	$\frac{1}{2}(0) + \frac{1}{3}y = 2$
$\frac{1}{2}x = 2$	$\frac{1}{3}y = 2$
$x = 4$	$y = 6$

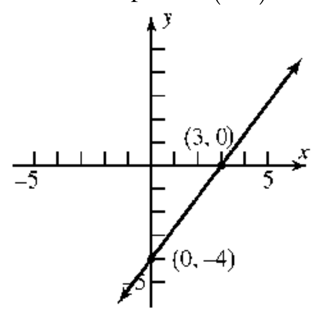
The intercepts are $(4, 0)$ and $(0, 6)$.



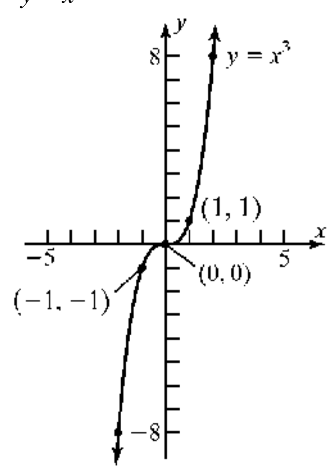
44. $\frac{1}{3}x - \frac{1}{4}y = 1$

x-intercept:	*y*-intercept:
$\frac{1}{3}x - \frac{1}{4}(0) = 1$	$\frac{1}{3}(0) - \frac{1}{4}y = 1$
$\frac{1}{3}x = 1$	$-\frac{1}{4}y = 1$
$x = 3$	$y = -4$

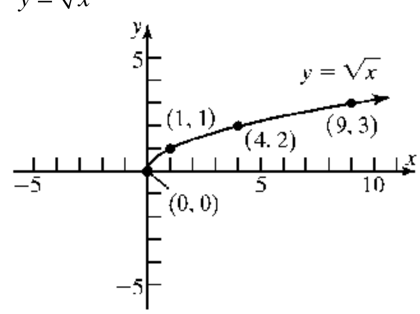
The intercepts are $(3, 0)$ and $(0, -4)$.



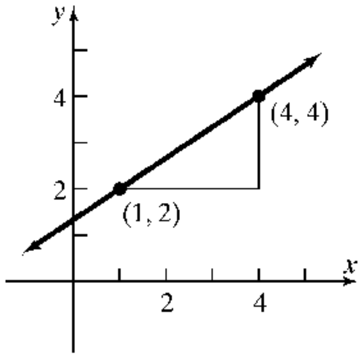
45. $y = x^3$



46. $y = \sqrt{x}$



47. slope = $\frac{2}{3}$, containing the point (1,2)



48. Find the distance between each pair of points.

$d_{A,B} = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$

$d_{B,C} = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$d_{A,C} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$

Since $AB = BC$, triangle ABC is isosceles.

49. Given the points $A = (-2, 0)$, $B = (-4, 4)$, and $C = (8, 5)$.

a. Find the distance between each pair of points.

$$d(A,B) = \sqrt{(-4-(-2))^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$d(B,C) = \sqrt{(8-(-4))^2 + (5-4)^2} = \sqrt{144+1} = \sqrt{145}$$

$$d(A,C) = \sqrt{(8-(-2))^2 + (5-0)^2} = \sqrt{100+25} = \sqrt{125} = 5\sqrt{5}$$

$$\left[d(A,B)\right]^2 + \left[d(A,C)\right]^2 = \left[d(B,C)\right]^2$$

$$\left(\sqrt{20}\right)^2 + \left(\sqrt{125}\right)^2 = \left(\sqrt{145}\right)^2$$

$$20 + 125 = 145$$

$$145 = 145$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

b. Find the slopes:

$$m_{AB} = \frac{4-0}{-4-(-2)} = \frac{4}{-2} = -2$$

$$m_{BC} = \frac{5-4}{8-(-4)} = \frac{1}{12}$$

$$m_{AC} = \frac{5-0}{8-(-2)} = \frac{5}{10} = \frac{1}{2}$$

Since $m_{AB} \cdot m_{AC} = -2 \cdot \frac{1}{2} = -1$, the sides AB and AC are perpendicular and the triangle is a right triangle.

50. Endpoints of the diameter are (–3, 2) and (5,–6). The center is at the midpoint of the diameter:

Center: $\left(\frac{-3+5}{2}, \frac{2+(-6)}{2}\right) = (1,-2)$

Radius: $r = \sqrt{(1-(-3))^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Equation: $(x-1)^2 + (y+2)^2 = \left(4\sqrt{2}\right)^2$

$(x-1)^2 + (y+2)^2 = 32$

51. slope of $\overline{AB} = \frac{1-5}{6-2} = -1$

slope of $\overline{AC} = \frac{-1-5}{8-2} = -1$

slope of $\overline{BC} = \frac{-1-1}{8-6} = -1$

Therefore, the points lie on a line.

52. $p = kB$

$854 = k(130{,}000)$

$k = \frac{854}{130{,}000} = \frac{427}{65{,}000}$

Therefore, we have the equation $p = \frac{427}{65{,}000}B$.

If $B = 165{,}000$, then

$p = \frac{427}{65{,}000}(165{,}000) = \$1083.92$.

53. $R = kg$

$46.67 = k(13)$

$k = \frac{46.67}{13} = 3.59$

Therefore, we have the equation $R = 3.59g$.

If $g = 11.2$, then $p = 3.59(11.2) \approx \$40.21$.

54. $w = \frac{k}{d^2}$

$200 = \frac{k}{3960^2}$

$k = (200)(3960^2) = 3,136,320,000$

Therefore, we have the equation $w = \frac{3,136,320,000}{d^2}$.

If $d = 3960 + 1 = 3961$ miles, then $w = \frac{3,136,320,000}{3961^2} \approx 199.9$ pounds.

55. $T^2 = ka^3$

$365^2 = (k)(93)^3$

$k = \frac{365^2}{93^3}$

Therefore, we have the equation $T^2 = \frac{365^2}{93^3}a^3$.

If T = 88 days, then

$88^2 = \left(\frac{365^2}{93^3}\right)(a)^3$

$a^3 = (88^2)\left(\frac{93^3}{365^2}\right)$

$a = \sqrt[3]{(88^2)\left(\frac{93^3}{365^2}\right)} \approx 36$ million miles

56. Answers will vary.

57. **a.** The graph of $x = 0$ is a vertical line passing through the origin. That is, $x = 0$ is the equation of the y-axis.

b. The grapy of $y = 0$ is a horizontal line passing through the origin. That is, $y = 0$ is the equation of the x-axis.

c. $x + y = 0$

$y = -x$

The graph of $x + y = 0$ is line passing through the origin with slope = –1.

d. $xy = 0$

$y = 0$ or $x = 0$

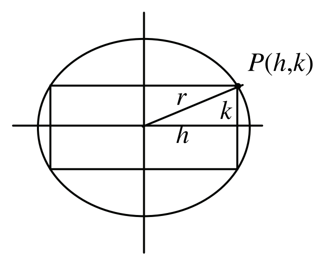
The graph of $xy = 0$ consists of the coordinate axes.

e. $x^2 + y^2 = 0$

$y = 0$ and $x = 0$

The graph of $x^2 + y^2 = 0$ is consists of the origin.

58. Set the axes so that the field's maximum dimension is along the x-axis.



Let $2h = width$, $2k = height$, therefore the point farthest from the origin has coordinates $P(h,k)$. So the distance from the origin to point P is $r = \sqrt{h^2 + k^2}$ = the radius of the circle .

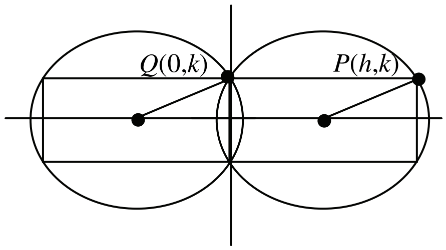
Using 1 sprinkler arm:

If we place the sprinkler at the origin, we get a circle with equation $x^2 + y^2 = r^2$, where $r = \sqrt{h^2 + k^2}$. So how much excess land is being watered ? The area of the field = $A_F = 4hk$.

The area of the circular water pattern $A_C = \pi r^2 = \pi\left(\sqrt{h^2 + k^2}\right)^2 = \pi\left(h^2 + k^2\right)$.

Therefore the amount of excess land being watered = $A_C - A_F = \pi\left(h^2 + k^2\right) - 4hk$.

Using 2 sprinkler arms:



We want to place the sprinklers so that they overlap as little as possible while watering the entire field. The equation of the circle with center on the positive x-axis that passes through the point $P(h,k)$ and $Q(0,k)$ is

$\left(x-\frac{h}{2}\right)^2 + y^2 = \sqrt{\frac{1}{4}h^2+k^2}$, since the center is $\left(\frac{h}{2},0\right)$, and the radius is $\sqrt{\frac{1}{4}h^2+k^2}$.

The case for the other side is similar. Thus, the sprinklers should have their centers at $\left(-\frac{h}{2},0\right)$ and $\left(\frac{h}{2},0\right)$, with the arm lengths set at $\sqrt{\frac{1}{4}h^2+k^2}$.

Each sprinkler waters the same area, so the total area watered is

$$A_C = 2\pi r^2 = 2\pi\left(\sqrt{\frac{1}{4}h^2+k^2}\right)^2 = 2\pi\left(\frac{1}{4}h^2+k^2\right).$$

The amount of excess land being watered is

$$A_C - A_F = 2\pi\left(\frac{1}{4}h^2+k^2\right) - 4hk \ .$$

Comparison:

In order to determine when to switch from 1 sprinkler to 2 sprinklers, we want to determine when 2 sprinklers water less excess land than 1 sprinkler waters. That is, we want to solve:

$$2\pi\left(\frac{1}{4}h^2+k^2\right) - 4hk < \pi\left(h^2+k^2\right) - 4hk \ .$$

$$2\pi\left(\frac{1}{4}h^2+k^2\right) - 4hk < \pi\left(h^2+k^2\right) - 4hk$$

$$\frac{\pi}{2}h^2 + 2\pi\ k^2 - 4hk < \pi\ h^2 + \ \pi\ k^2 - 4hk$$

$$\frac{\pi}{2}h^2 + 2\pi\ k^2 < \pi\ h^2 + \ \pi\ k^2$$

$$\frac{1}{2}h^2 + 2\ k^2 < h^2 + \ k^2$$

$$k^2 < \frac{1}{2}h^2$$

$$k < \sqrt{\frac{1}{2}}h$$

$$h > \sqrt{2}k$$

So 2 sprinklers is the better choice when the longer dimension of the rectangle exceeds the shorter dimension by a factor of more than $\sqrt{2} \approx 1.414$.

Chapter 2 Test

1. $d(P_1,P_2) = \sqrt{(5-(-1))^2 + (-1-3)^2}$

$= \sqrt{6^2 + (-4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

2. The coordinates of the midpoint are:

$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+5}{2}, \frac{3+(-1)}{2}\right)$

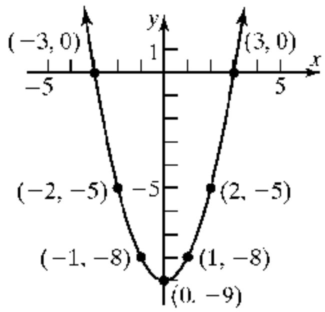
$= \left(\frac{4}{2}, \frac{2}{2}\right)$

$= (2,1)$

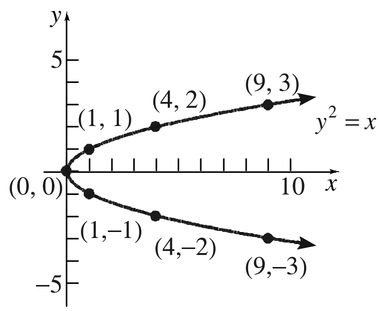
3. **a.** $m = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-3}{5-(-1)} = \frac{-4}{6} = -\frac{2}{3}$

b. If x increases by 3 units, y will decrease by 2 units.

4. $y = x^2 - 9$



5. $y^2 = x$



6. $x^2 + y = 9$

x-intercepts: $x^2 + 0 = 9$, $x^2 = 9$, $x = \pm 3$

y-intercept: $(0)^2 + y = 9$, $y = 9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test *x*-axis symmetry: Let $y = -y$

$x^2 + (-y) = 9$

$x^2 - y = 9$ different

Test *y*-axis symmetry: Let $x = -x$

$(-x)^2 + y = 9$

$x^2 + y = 9$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 + (-y) = 9$

$x^2 - y = 9$ different

Therefore, the graph will have *y*-axis symmetry.

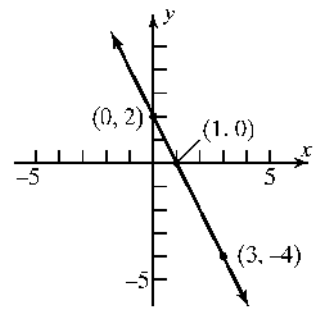
7. Slope $= -2$; containing $(3,-4)$

$y - y_1 = m(x - x_1)$

$y - (-4) = -2(x - 3)$

$y + 4 = -2x + 6$

$y = -2x + 2$



8. $(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y-(-3))^2 = 5^2$

$(x-4)^2 + (y+3)^2 = 25$

General form: $(x-4)^2 + (y+3)^2 = 25$

$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 8x + 6y = 0$

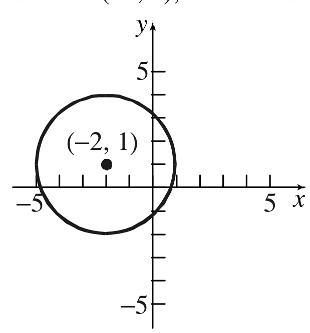
9. $x^2 + y^2 + 4x - 2y - 4 = 0$

$x^2 + 4x + y^2 - 2y = 4$

$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 4 + 1$

$(x+2)^2 + (y-1)^2 = 3^2$

Center: (–2, 1); Radius = 3



10. $2x+3y=6$

$$3y=-2x+6$$

$$y=-\frac{2}{3}x+2$$

<u>Parallel line</u>
Any line parallel to $2x+3y=6$ has slope $m=-\frac{2}{3}$. The line contains $(1,-1)$:

$$y-y_1=m(x-x_1)$$

$$y-(-1)=-\frac{2}{3}(x-1)$$

$$y+1=-\frac{2}{3}x+\frac{2}{3}$$

$$y=-\frac{2}{3}x-\frac{1}{3}$$

<u>Perpendicular line</u>
Any line perpendicular to $2x+3y=6$ has slope $m=\frac{3}{2}$. The line contains $(0,3)$:

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{3}{2}(x-0)$$

$$y-3=\frac{3}{2}x$$

$$y=\frac{3}{2}x+3$$

11. Let R = the resistance, l = length, and r = radius. Then $R=k\cdot\frac{l}{r^2}$. Now, R = 10 ohms, when l = 50 feet and $r=6\times10^{-3}$ inch, so

$$10=k\cdot\frac{50}{\left(6\times10^{-3}\right)^2}$$

$$k=10\cdot\frac{\left(6\times10^{-3}\right)^2}{50}=7.2\times10^{-6}$$

Therefore, we have the equation

$$R=\left(7.2\times10^{-6}\right)\frac{l}{r^2}.$$

If $l=100$ feet and $r=7\times10^{-3}$ inch, then

$$R=\left(7.2\times10^{-6}\right)\frac{100}{\left(7\times10^{-3}\right)^2}\approx 14.69 \text{ ohms.}$$

Chapter 2 Cumulative Review

1. $3x-5=0$

$$3x=5$$

$$x=\frac{5}{3}$$

The solution set is $\left\{\frac{5}{3}\right\}$.

2. $x^2-x-12=0$

$$(x-4)(x+3)=0$$

$$x=4 \text{ or } x=-3$$

The solution set is $\{-3,4\}$.

3. $2x^2-5x-3=0$

$$(2x+1)(x-3)=0$$

$$x=-\frac{1}{2} \text{ or } x=3$$

The solution set is $\left\{-\frac{1}{2},3\right\}$.

4. $x^2-2x-2=0$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$$

$$=\frac{2\pm\sqrt{4+8}}{2}$$

$$=\frac{2\pm\sqrt{12}}{2}$$

$$=\frac{2\pm2\sqrt{3}}{2}$$

$$=1\pm\sqrt{3}$$

The solution set is $\left\{1-\sqrt{3},1+\sqrt{3}\right\}$.

5. $x^2+2x+5=0$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(5)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{4-20}}{2}$$

$$=\frac{-2\pm\sqrt{-16}}{2}$$

No real solutions

6. $\sqrt{2x+1} = 3$

$\left(\sqrt{2x+1}\right)^2 = 3^2$

$2x+1 = 9$

$2x = 8$

$x = 4$

Check: $\sqrt{2(4)+1} = 3?$

$\sqrt{9} = 3?$

$3 = 3$ True

The solution set is $\{4\}$.

7. $|x-2| = 1$

$x-2 = 1$ or $x-2 = -1$

$x = 3$ $\quad x = 1$

The solution set is $\{1, 3\}$.

8. $\sqrt{x^2+4x} = 2$

$\left(\sqrt{x^2+4x}\right)^2 = 2^2$

$x^2+4x = 4$

$x^2+4x-4 = 0$

$$x = \frac{-4 \pm \sqrt{4^2-4(1)(-4)}}{2(1)} = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$= \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

Check $x = -2+2\sqrt{2}$:

$\sqrt{\left(-2+2\sqrt{2}\right)^2 + 4\left(-2+2\sqrt{2}\right)} = 2?$

$\sqrt{4-8\sqrt{2}+8-8+8\sqrt{2}} = 2?$

$\sqrt{4} = 2$ True

Check $x = -2-2\sqrt{2}$:

$\sqrt{\left(-2-2\sqrt{2}\right)^2 + 4\left(-2-2\sqrt{2}\right)} = 2?$

$\sqrt{4+8\sqrt{2}+8-8-8\sqrt{2}} = 2?$

$\sqrt{4} = 2$ True

The solution set is $\left\{-2-2\sqrt{2}, -2+2\sqrt{2}\right\}$.

9. $x^2 = -9$

$x = \pm\sqrt{-9}$

$x = \pm 3i$

The solution set is $\{-3i, 3i\}$.

10. $x^2-2x+5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{4-20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

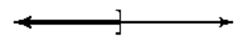
The solution set is $\{1-2i, 1+2i\}$.

11. $2x-3 \le 7$

$2x \le 10$

$x \le 5$

$\{x \mid x \le 5\}$ or $(-\infty, 5]$



12. $-1 < x+4 < 5$

$-5 < x < 1$

$\{x \mid -5 < x < 1\}$ or $(-5, 1)$

13. $|x-2| \le 1$

$-1 \le x-2 \le 1$

$1 \le x \le 3$

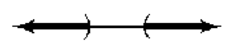
$\{x \mid 1 \le x \le 3\}$ or $[1, 3]$

14. $|2+x| > 3$

$2+x < -3$ or $2+x > 3$

$x < -5$ or $x > 1$

$\{x \mid x < -5 \text{ or } x > 1\}$ or $(-\infty, -5) \cup (1, \infty)$



15. $d(P,Q) = \sqrt{(-1-4)^2 + (3-(-2))^2}$

$= \sqrt{(-5)^2 + (5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50} = 5\sqrt{2}$

$$\text{Midpoint} = \left(\frac{-1+4}{2}, \frac{3+(-2)}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$$

16. $y = x^3 - 3x + 1$

a. $(-2,-1)$:

$(-2)^3 - (3)(-2) + 1 = -8 + 6 + 1 = -1$

$(-2,-1)$ is on the graph.

b. $(2,3)$:

$(2)^3 - (3)(2) + 1 = 8 - 6 + 1 = 3$

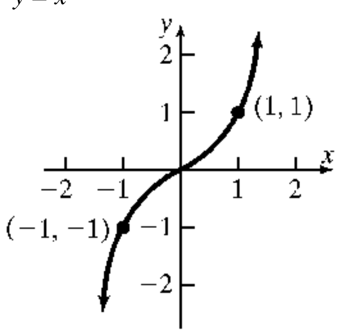
$(2,3)$ is on the graph.

c. $(3,1)$:

$(3)^3 - (3)(3) + 1 = 27 - 9 + 1 = 19 \neq 1$

$(3,1)$ is not on the graph.

17. $y = x^3$



18. The points (–1,4) and (2,–2) are on the line.

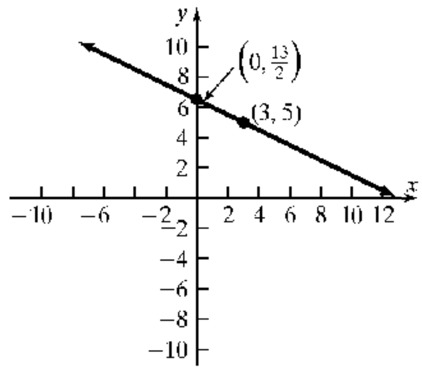
$$\text{Slope} = \frac{-2-4}{2-(-1)} = \frac{-6}{3} = -2$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -2\left(x - (-1)\right)$$
$$y - 4 = -2(x + 1)$$
$$y = -2x - 2 + 4$$
$$y = -2x + 2$$

19. Perpendicular to $y = 2x + 1$; Contains $(3,5)$

$$\text{Slope of perpendicular} = -\frac{1}{2}$$

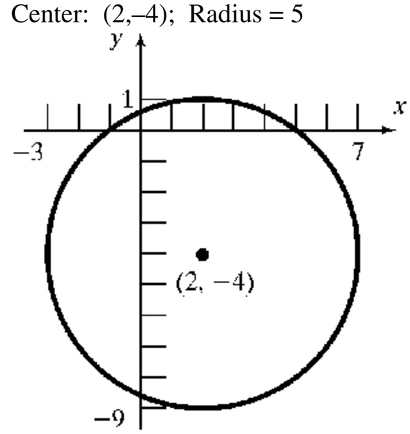
$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{1}{2}(x - 3)$$
$$y - 5 = -\frac{1}{2}x + \frac{3}{2}$$
$$y = -\frac{1}{2}x + \frac{13}{2}$$



20.

$$x^2 + y^2 - 4x + 8y - 5 = 0$$
$$x^2 - 4x + y^2 + 8y = 5$$
$$(x^2 - 4x + 4) + (y^2 + 8y + 16) = 5 + 4 + 16$$
$$(x-2)^2 + (y+4)^2 = 25$$
$$(x-2)^2 + (y+4)^2 = 5^2$$

Center: (2,–4); Radius = 5



Chapter 2 Projects

1. <u>Men</u>: Let $(x_1, y_1) = (1992, 2.22)$ and $(x_2, y_2) = (1996, 2.21)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.21 - 2.22}{1996 - 1992} = \frac{-0.01}{4} = -0.0025$$

$$y - y_1 = m(x - x_1)$$
$$y - 2.22 = -0.0025(x - 1992)$$
$$y - 2.22 = -0.0025x + 4.98$$
$$y = -0.0025x + 7.20$$

Women: Let $(x_1, y_1) = (1992, 2.54)$ and $(x_2, y_2) = (1996, 2.43)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.43 - 2.54}{1996 - 1992} = \frac{-0.11}{4} = -0.0275$$

$$y - y_1 = m(x - x_1)$$
$$y - 2.54 = -0.0275(x - 1992)$$
$$y - 2.54 = -0.0275x + 54.78$$
$$y = -0.0275x + 57.32$$

2. Men: The slope $m = -0.0025$ indicates that the winning times for men in the Olympic marathon are decreasing at an average rate of 0.0025 hour per year.

   Women: The slope $m = -0.0275$ indicates that the winning times for women in the Olympic marathon are decreasing at an average rate of 0.0275 hour per year.

   The y-intercepts do not have reasonable interpretations. In this case, the y-intercepts would indicate the winning times of the marathons in the year 0, which is not reasonable because it is too far away from the data on which our equations are based.

3. Men: If $x = 2004$, then $y = -0.0025(2004) + 7.20 = 2.19$ hours. This compares reasonably well to the actual result of 2.18 hours.

   Women: If $x = 2004$, then $y = -0.0275(2004) + 57.32 = 2.21$ hours. This does not compare well to the actual result of 2.44 hours.

4. **(1)** Men: Let $(x_1, y_1) = (1996, 2.21)$ and $(x_2, y_2) = (2000, 2.17)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.17 - 2.21}{2000 - 1996} = \frac{-0.04}{4} = -0.01$$

$$y - y_1 = m(x - x_1)$$
$$y - 2.21 = -0.01(x - 1996)$$
$$y - 2.21 = -0.01x + 19.96$$
$$y = -0.01x + 22.17$$

Women: Let $(x_1, y_1) = (1996, 2.43)$ and $(x_2, y_2) = (2000, 2.39)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.39 - 2.43}{2000 - 1996} = \frac{-0.04}{4} = -0.01$$

$$y - y_1 = m(x - x_1)$$
$$y - 2.39 = -0.01(x - 1996)$$
$$y - 2.39 = -0.01x + 19.96$$
$$y = -0.01x + 22.35$$

(2) Men: The slope $m = -0.01$ indicates that the winning times for men in the Olympic marathon are decreasing at a rate of 0.01 hour per year.

Women: The slope $m = -0.01$ indicates that the winning times for women in the Olympic marathon are decreasing at a rate of 0.01 hour per year.

The y-intercepts do not have reasonable interpretations. In this case, the y-intercepts would indicate the winning times of the marathons in the year 0, which is not reasonable because it is too far away from the data on which our equations are based.

(3) Men: If $x = 2004$, then $y = -0.01(2004) + 22.17 = 2.13$ hours. This prediction is not extremely accurate, but it does compare reasonably well to the actual result of 2.18 hours.

Women: If $x = 2004$, then $y = -0.01(2004) + 22.35 = 2.31$ hours. This does not compare well to the actual result of 2.44 hours.

5. No. The year 2104 is too far away from the data on which our equations are based.

6. Answers will vary.