

INSTRUCTOR'S
RESOURCE MANUAL

DEVELOPMENTAL
MATHEMATICS:
COLLEGE MATHEMATICS AND
INTRODUCTORY ALGEBRA
TENTH EDITION

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CONTENTS

Introduction iv

Available Supplements v

Collaborative Learning Activities CLA-1

Mini-Lectures ML-1

Mini-Lecture Graph Answers MLGA-1

Course Diagnostic CD-1

Answer Key for Course Diagnostic CDA-1

Chapter Tests CT-1

Answer Keys for Chapter Tests CTA-1

Final Examinations FE-1

Answer Keys for Final Examinations FEA-1

INTRODUCTION

Dear Faculty:

The Bittinger book team at Pearson knows that whether you are teaching this course for the first time or the tenth time, you will face many challenges, including how to prepare for class, how to make the most effective use of class time, how to present the material to your students in a manner that makes sense to them, and how best to assess your students.

This *Instructor's Resource Manual* is designed to make your job easier. Inside are words of advice from experienced instructors, general and content-specific teaching tips, a list of the objectives covered within, and descriptions of available student and instructor supplements.

It is also important to know that you have a very valuable resource available to you in your Pearson sales representative. If you do not know your representative, you can locate him/her by logging on to www.pearsonhighered.com/relocator and typing in your zip code. Please feel free to contact your representative if you have any questions relating to our text or if you need additional supplements.

We know teaching this course can be challenging. We hope that this, and the other resources we have provided, will help to minimize the amount of time it takes you to meet those challenges.

Good luck in your endeavors!

The Bittinger book team

AVAILABLE SUPPLEMENTS

Student Supplements	Instructor Supplements
<p>New! MyMathGuide: Notes, Practice, and Video Path workbook This objective-based workbook for guided, hands-on learning offers vocabulary, skill, and concept review—along with problem-solving practice—with space to show work and write notes. Incorporated in the Learning Path in MyLab Math, <i>MyMathGuide</i> can be use together with the To-the-Point Objective Video program, instructor lectures, and the textbook. Instructors can assign To-the-Point Objective Videos in MyLab Math in conjunction with the <i>MyMathGuide</i> workbook.</p> <p>Student’s Solutions Manual By Judith Henn Contains completely worked-out annotated solutions for all the odd-numbered exercises in the text. Also includes fully worked-out annotated solutions for all the exercises (odd- and even-numbered) in the Mid-Chapter Reviews, the Summary and Reviews, the Chapter Tests, and the Cumulative Reviews.</p> <p>Chapter Test Prep Videos Chapter Tests can serve as practice tests to help you study. Watch instructors work through step-by-step solutions to all the Chapter Test exercises from the textbook. Chapter Test Prep videos are available on YouTube (search using author and book title) and in MyLab Math.</p> <p>InterAct Math Tutorial Website (www.interactmath.com) Get practice and tutorial help online! This interactive tutorial website provides algorithmically generated practice exercises that correlate directly to the exercises in the textbook. Students can retry an exercise as many times as they like with new values each time for unlimited practice and mastery. Every exercise is accompanied by an interactive guided solution that provides helpful feedback for incorrect answers, and students can also view a worked-out sample problem that steps them through an exercise similar to the one they’re working on.</p>	<p>Annotated Instructor’s Edition Includes answers to all exercises printed in blue on the same page as the exercises. Also includes the student answer section for easy reference.</p> <p>Instructor’s Solutions Manual (download only) By Judith Henn Contains brief solutions to the even-numbered exercises in the exercise sets. Also includes fully worked-out annotated solutions for all the exercises (odd- and even-numbered) in the Mid-Chapter Reviews, the Summary and Reviews, the Chapter Tests, and Cumulative Reviews.</p> <p>PowerPoint Lecture Slides (download only) Present key concepts and definitions from the text.</p> <p>Instructor’s Resource Manual with Tests and Mini-Lectures (download only)</p> <ul style="list-style-type: none">• Features resources and teaching tips designed to help both new and adjunct faculty with course preparation and classroom management.• Includes a mini-lecture for each section of the text with objectives, key examples, and teaching tips.• Additional resources include collaborative learning activities, transition guide, and transparency masters.

For Students and Instructors

MyLab[®] Math Online Course (access code required)

MyLab Math from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy-to-use format. MyLab Math delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future.

Additional Media Supplements

MathXL[®] Online Course (access code required)

Math XL[®] is a powerful online homework, tutorial, and assessment system that accompanies Pearson Education's textbooks in mathematics or statistics.

With MathXL, instructors can

- create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- create and assign their own online exercises and import TestGen tests for added flexibility.
- maintain records of all student work tracked in MathXL's online gradebook.

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- take chapter tests in MathXL and receive personalized study plans based on their test results.
- use the study plan to link directly to tutorial exercises for the objectives they need to study and retest.
- access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com or contact your Pearson sales representative.

TestGen[®] (www.pearsoned.com/testgen) enables instructors to build, edit, and print tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson Education's online catalog.

PowerPoint[®] Lecture Slides present key concepts and definitions from the text. Slides are available to download from within MyLab Math and from Pearson Education's online catalog.

COLLABORATIVE LEARNING ACTIVITIES

Name

Section

Date

Activity 1.4 Solving equations in pairs.

Focus	Solving linear equations
Time	10–20 minutes
Group size	2
Materials	Sets of twenty 3×5 cards with linear equations like those in section 1.4 of the text written on one side.
Background	This activity will give you practice solving linear equations and checking solutions.
Instructor notes	The equations written on the 3×5 cards will be solvable in one step, like problems 5 – 56 in section 1.4. Write one equation per card. Make duplicate sets, so that each group may have a set of twenty equations.

1. The person with the shortest first name will divide the 3×5 cards into two equal piles, equation side down. Each person should have the same number of cards in his or her stack.
2. When the instructor gives the signal to begin, each person will pick up the top card on his or her pile and solve the equation, showing the work on the card.
3. Exchange cards with your partner and each of you check the solution your partner found in the original equation on that card. Show your work.
4. If the solution checks, put the card face up in a “finished” pile.
5. If the solution does not check, try to find the error and, working with your partner, correct it. When you are in agreement and the solution checks, put the card in the “finished” pile and pick up the next two cards.
6. Continue steps 2 through 5 until all equations have been solved correctly.

Conclusion	Participating in this activity gives you practice in solving linear equations. You will be solving equations throughout the remainder of this course and in future mathematics courses. The procedures you learn will prove useful to you whenever you are solving equations.
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Activity 1.5 Make a budget for a road trip to you favorite destination.

Focus	Problem solving and estimation
Time	20–30 minutes
Group size	3–4
Materials	State and local highway maps for each group, calculators (optional)
Background	Planning a road trip involves several mathematical computations. For instance, the total distance to be traveled and the estimated cost for gas can be calculated using the concepts learned in this chapter.

1. Before you begin your calculations, select a destination for a road trip you could take on long weekend. Decide on one destination for your group.

Origin: _____

Destination: _____

2. Using the appropriate state and local highway map(s), highlight the route you would take to get to your destination and back home again. Calculate the total distance you would need to drive. Round this distance to the nearest hundred miles.

Total distance: _____

Estimated distance: _____

3. Estimate the gallons of gas you would need for your trip. Use the miles per gallon (mpg) rating on one of your group member’s vehicle. Then calculate the total cost of the gas. Use the price per gallon of gas in your area.

Gallons of gas needed: _____

Total cost for gas: _____

4. Now decide how many days and nights it would take to complete the trip. Then calculate the cost for the accommodations.

Days of travel: _____

Number of nights accommodation: _____

Total cost for accommodations: _____

5. Based on the days of travel, calculate how many meals you would need to eat during the trip. Then calculate the cost of the meals for all the people on this trip.

Number of meals per person: _____

Total number of meals: _____

Cost for meals: _____

6. Summarize your estimated costs below. Include a reasonable figure for the cost of miscellaneous items. These might include the cost of admission tickets, souvenirs, parking, and tolls.

Item	Estimated Cost
Gas:	
Accommodation:	
Meals	
Miscellaneous:	
Total:	

Conclusion	As you can see, planning a budget for a road trip involves the operations of addition, subtraction, multiplication, and division, as well as estimation. Use the steps given in this activity to plan a road trip for yourself and family or friends.
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Activity 1.6 Use the order of operations as a group to simplify expressions.

Focus	Order of operations
Time	20–30 minutes
Group size	3
Background	Simplifying expressions using the rules for order of operations can be quite confusing for complicated expressions. Learning to simplify expressions as a group will help clarify the process.

Rules for Order of Operations

		Do all calculations within parentheses before operations outside.
	E	Evaluate all exponential expressions.
	MD	Do all multiplications and divisions in order from left to right.
	AS	Do all additions and subtractions in order from left to right.

1. Before you begin simplifying expressions, study the rules for order of operations above. Assign each group member to one of the steps listed. Write the name of the group member next to his or her assigned task in the table above. Note that the first step (calculations within parentheses) is not assigned. All group members will do this step together.

2. Now you are ready to simplify expressions as a group. Analyze the expression together and decide on the first step. If there are parentheses, decide whether the expression inside the parentheses needs to be simplified. Following the order of operations, **E** will perform his or her task before **MD**, and **MD** will perform his or her task before **AS**.

Practice with the example on the next page. (This is Example 10, Section 1.6 in your textbook.) The first step has been done for you: Subtract inside the parentheses. **AS** will do this step, writing “**AS**” in the left box, and writing the new expression below the original expression.

Continue simplifying the expression by passing the problem to the appropriate group member for the next step. When you are done, compare your steps to those in Example 10, Section 1.6 in your textbook. If there are any discrepancies, discuss

them within your group. Compare your result with the other groups. Are they the same? Discuss any differences with the other groups.

Example 10, Section 1.6

	$4^2 \div (10 - 9 + 1)^3 \cdot 3 - 5$
AS	$4^2 \div (1 + 1)^3 \cdot 3 - 5$

- Once you understand the process, choose an expression from Exercise Set 1.6 in your textbook to simplify as a group. Use the table on the next page to organize your work. Make as many copies as you need. Alternatively, you can draw the table on a blank sheet of paper.

Do as many problems as you can in the time allotted. Reassign tasks (E, MD, AS) to different members of the group. Do this at least twice so that each member of the group has a turn performing each of the tasks. Make sure you choose at least one of the more complicated expressions from Exercises 59 – 68.

Conclusion	This activity should help you gain a better understanding of the rules for order of operations. You can also use this group method when you encounter the order of operations in Sections 2.6, 3.4, and 7.8.
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Original expression _____

Name

Section

Date

Activity 1.7 Find all the prime numbers less than 100, using the Sieve of Eratosthenes.

Focus	Prime and composite numbers
Time	10–15 minutes
Group size	2
Material	Colored pencils (optional)
Background	One of the methods for finding prime numbers was developed around 200 BC by a mathematician named Eratosthenes. He used the process of elimination to “sift” out the composite numbers, leaving only prime numbers. His method became known as the Sieve of Eratosthenes.

1. In Section 1.7 of your textbook, a prime number is defined as a natural number that has exactly two different factors, itself and 1. For example, the number 7 is prime because it has only the factors 1 and 7. The number 14, on the other hand, is not prime because 7 is a factor of 14. Looking at the definition from another point of view, any number that is a multiple of another number (besides 1) will not be prime. In the example above, 14 is a multiple of 7, so 14 is not prime.

In this activity, you will cross off all multiples of prime numbers from a grid of numbers. When you are done, the remaining numbers will be prime.

2. Look at the grid on the next page. The number 1 has already been crossed off, as it is not a prime number. The smallest number that is not crossed off is 2. Begin by circling the number 2 on the grid. Then, list the first 10 multiples of 2 in the space below:

Now, cross off these numbers from the grid. You may want to use a colored pencil to cross off the numbers. Continue crossing off multiples of 2 until you reach the end of the grid.

3. Next, look for the smallest number that is not crossed off and circle it. This is the next prime number. List the first 10 multiples of this number in the space below:

Cross off these numbers from the grid. Continue, as before, crossing off multiples of the number until you reach the end of the grid.

X	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Repeat step 3 until all multiples are crossed off. The circled numbers are the prime numbers less than 100. Write the list of circled numbers in the space below:

- Compare this list with the table of primes given in section 1.7 of your textbook. Are there any differences between the lists? If there are, check your grid to see if you crossed off all multiples. Check also that you did not accidentally cross off a number that is not a multiple.

Conclusion	The Sieve of Eratosthenes can be used anytime you need to list the first few prime numbers. For example, if you need all the prime numbers up to 50, make a list of the numbers from 1 to 50, and start crossing out the multiples of 2, 3, 5, etc.
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Activity 1.8 Use the divisibility rules and properties of numbers to discover an unknown number.

Focus	Rules for divisibility, place value
Time	20–30 minutes
Group size	2
Background	The rules for divisibility given in Section 1.8 of your textbook provide you with fast ways of determining whether numbers are divisible by 2, 3, 4, 5, 6, 8, 9, and 10. This activity will provide practice with these rules, as well as experience in problem solving.
Instructor notes	In step 4, show Puzzles A and B, revealing clues one at a time. You can find more puzzles in the book <i>Logic Number Problems</i> , available from Dale Seymour Publications.

For your convenience, the divisibility rules from Section 1.8 are repeated here.

2	A number is divisible by 2 (is even) if it has a ones digit of 0, 2, 4, 6, or 8.
3	A number is divisible by 3 if the sum of its digits is divisible by 3.
4	A number is divisible by 4 if the number named by its last two digits is divisible by 4.
5	A number is divisible by 5 if its ones digit is 0 or 5.
6	A number is divisible by 6 if its ones digit is 0, 2, 4, 6, or 8 (is even) and the sum of its digits is divisible by 3.
8	A number is divisible by 8 if the number named by its last three digits is divisible by 8.
9	A number is divisible by 9 if the sum of its digits is divisible by 9.
10	A number is divisible by 10 if its ones digit is 0.

- Each puzzle in this activity gives you clues to the value of an unknown number. The objective is to determine the unknown number by using the fewest number of clues. The clues will be given to you one at a time.
- First, practice on the following set of clues. Read the clues one at a time, using a sheet of paper to cover up the clues further down.

Clue		Possible solution(s)	Reasoning
1	It is a 3-digit number.	___ ___ ___	Write one blank for each digit.
2	It is divisible by 5.	___ ___ 0 ___ ___ 5	To be divisible by 5, the last digit must be 0 or 5.
3	It is an even number.	___ ___ 0	The last digit must be 0, 2, 4, 6, or 8.
4	It is less than 400.	3 ___ 0 2 ___ 0 1 ___ 0	The hundreds digit must be less than 4.
5	Each digit is different.	3 2 0	No digit can be repeated. The tens digit must be 1 or higher, and the hundreds digit must be 2 or higher.
6	Its tens digit is greater than its ones digit.	3 1 0 2 1 0	
7	Its hundreds digit is greater than its tens digit.		
8	It is divisible by 3.	2 1 0	The sum of its digits must be divisible by 3.
9	It has only one odd digit.		These clues confirm that the number is 210. They are actually not needed to solve the puzzle.
10	Its tens digit is 1.		

Notice that some clues must be considered together (clues 5, 6, and 7), and that only the first 8 clues are needed to solve this puzzle.

3. Here's another puzzle to practice on. One group member writes down the possible solutions, as was done in the example on the previous page. Use complete sentences when writing the reasons for each possible solution. Read the clues one at a time, using a sheet of paper to cover up the clues further down.

	Clue	Possible solution(s)	Reasoning
1	It is a 3-digit number.		
2	It is an odd number.		
3	One of the digits is 7.		
4	It is divisible by 5.		
5	It is less than 700.		
6	It has no even digits.		
7	It is divisible by 3.		
8	It is greater than 200.		
9	Each digit is different.		
10	It is a multiple of 25.		

When you are done, compare your group's result with the results of the other groups in your class. How many clues did your group need to solve this puzzle? Could you have determined the unknown number with fewer clues? Did you use the remaining clues (if any) to check your answer?

4. Now, let's add a little competition to the problem-solving process. Each group will work as a team to solve a puzzle. Take turns, so each group member has a chance to do the writing. Your instructor will reveal the clues one at a time. The goal is to be the first group to correctly deduce the unknown number by using the fewest number of clues. Your instructor will discuss the scoring scheme; alternatively, the class can propose a scheme that is acceptable to all. The scoring scheme should take into account the correctness of the number, the penalty for a wrong number, the number of clues used, and the penalty for using more clues than needed.

Conclusion	This activity should help you gain experience in applying the divisibility rules. As a side benefit, the problem solving techniques used in solving the puzzles will be useful in solving the applications in your textbook.
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Puzzle A

1	It is a 3-digit number
2	It is divisible by 5
3	It is an odd number
4	Each of its digits is different
5	Its tens digit is less than its ones digit
6	Its hundreds digit is less than its tens digit
7	It is greater than 200
8	It is divisible by 3
9	It has two odd digits
10	Its tens digit is 4

Puzzle B

1	It is a 3-digit number
2	It is divisible by 5
3	Its hundreds digit is 8
4	It is divisible by 3
5	Its tens digit is less than its ones digit
6	None of its digits are repeated
7	The sum of two of its digits is 10
8	It has only one odd digit
9	It is divisible by 11
10	Its tens digit is 2

Name

Section

Date

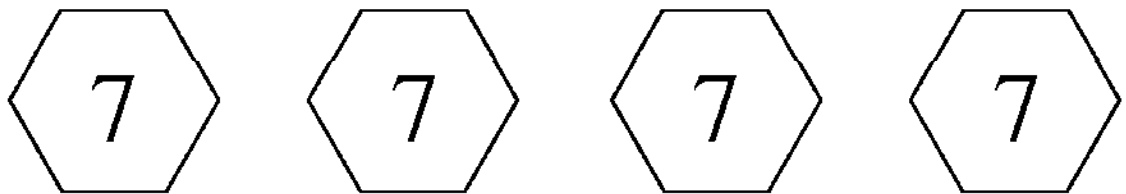
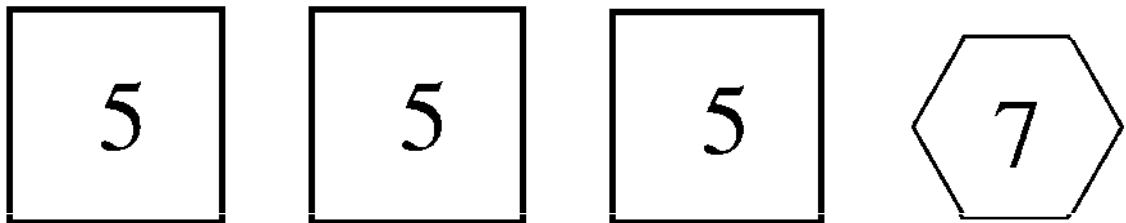
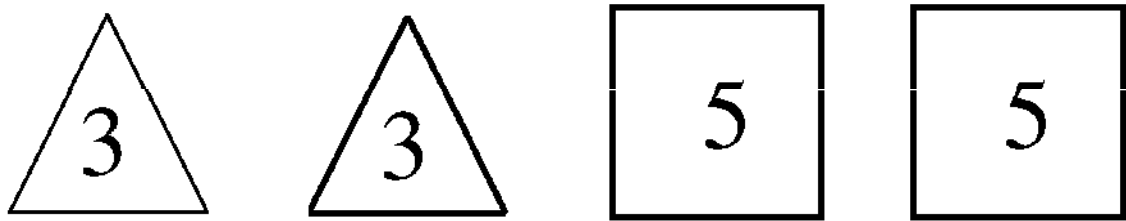
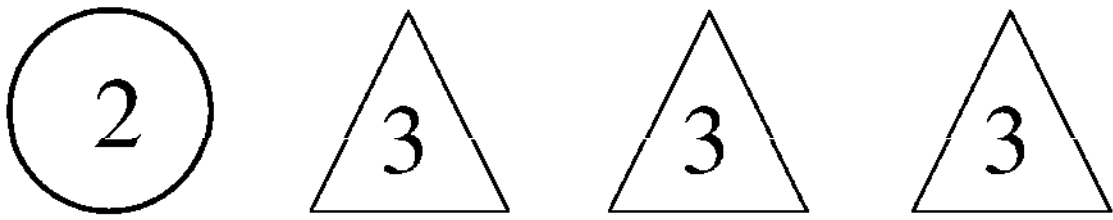
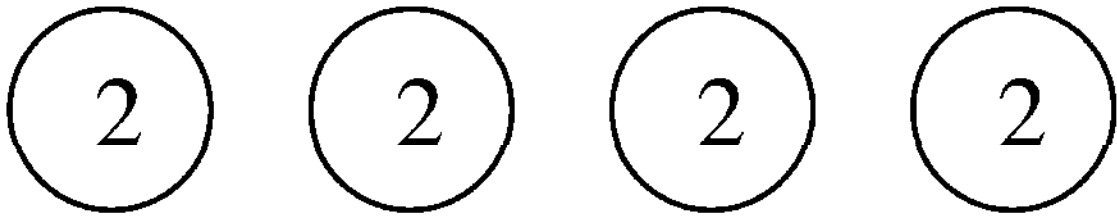
Activity 1.9 Find the least common multiple of two or more numbers using shaped markers.

Focus	Least common multiples
Time	20–30 minutes
Group size	2
Background	The textbook describes two methods for finding the least common multiple (LCM) of a set of numbers: using multiples and using factorizations. To find the LCM using factorizations, first, find the prime factorization of each number; then, create a product of factors, using each factor the greatest number of times it occurs in any one factorization. The second part of the factorization method requires the creation of a product of factors. We will see how these factors are chosen by using shaped markers to represent the factorizations. This visualization should give you a clearer picture of the process.
Instructor notes	Copy the next page on card stock, and cut out the markers. Each group will need one set of markers. You can also purchase sets of markers (also called pattern blocks).

1. Study the set of markers; notice that each type of marker represents a different prime number, as follows.

Marker	Prime number
Circle	2
Triangle	3
Square	5
Hexagon	7

For the first part of this activity, use a restricted subset of markers consisting of 3 circles, 3 triangles, and 3 squares.



Use the restricted set of markers to represent each of the following prime factorizations. You will need to place all used markers back in the subset before you work on the next prime factorization. Take turns selecting the markers, so each group member gets to practice this step.

$$90 = \overline{2} \cdot \overline{3} \cdot \overline{3} \cdot \overline{5}$$

$$150 = \overline{2} \cdot \overline{3} \cdot \overline{5} \cdot \overline{5}$$

Notice that by replacing markers back into the subset, you were able to reuse some markers. Therefore, it may not have been necessary to have 3 of each kind of marker. Experiment with your partner to find the minimum number of each marker needed to represent the prime factorizations given above. Remember that you do not need to represent the factorizations at the same time. List the minimum set of markers below.

- Now, use the full set of markers and practice finding the minimum set of markers needed for each of the following prime factorizations. Represent the minimum set with markers, and also write the prime factorization of the minimum set. Each group member should do one problem individually, then exchange papers and check each other's work.

Problem	Minimum set of markers	Prime factorization of minimum set
A. $18 = 2 \cdot 3 \cdot 3$ $30 = 2 \cdot 3 \cdot 5$		

<p>B. $75 = 3 \cdot 5 \cdot 5$ $30 = 2 \cdot 5 \cdot 7$</p>		
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6. The minimum set of markers needed for a group of prime factorizations is the least common multiple (LCM) of the group of numbers. Find the LCM of the following groups. You may use the markers to help you find the LCM.


Problem	LCM
<p>A. 50 75</p>	
<p>B. 24 18</p>	

<p>Conclusion</p>	<p>The factorization method for finding least common multiples can be visualized using prime number markers. The minimum set of markers needed to represent the prime factorization of each number will give the LCM of the numbers.</p>
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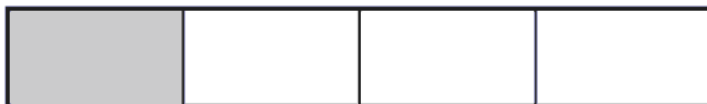
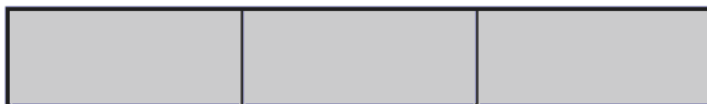
Activity 2.1 Use fraction bars to represent equivalent fractions.

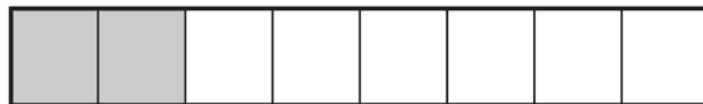
Focus	Equivalent fractions
Time	10–15 minutes
Group size	3
Background	Equivalent fractions are used extensively when adding, subtracting, and simplifying fractions. In Section 2.1 of your textbook, the process of multiplying by one is used to find equivalent fractions, and to simplify fractions. This activity will give you a better understanding of these concepts.
Instructor notes	Each group will need one set of bars from the next three pages. Copy the pages on card stock, and cut along the heavy outline of each bar; do not cut within each fraction bar. You can also purchase fraction bars.

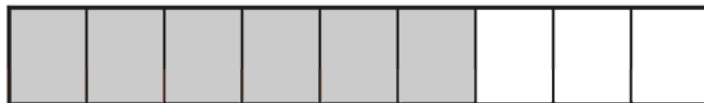
- Two fractions are equivalent if they represent the same number. For example, $\frac{2}{3}$ and $\frac{4}{6}$ are two names for the same number. We will use fraction bars to show how equivalent fractions can be represented visually. Take your group's set of fraction bars and mix them up. One group member selects the bar that represents $\frac{2}{3}$, and places it in the box below.

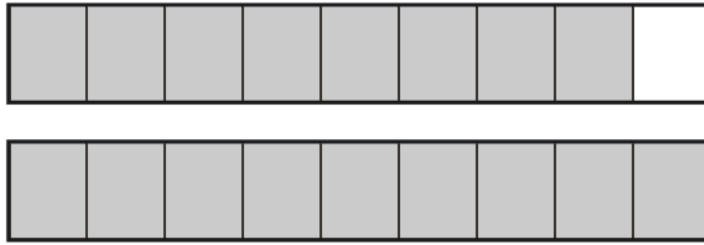
 $\frac{2}{3}$  $\frac{4}{6}$ 

Taking turns, the next group member selects the fraction bar that represents $\frac{4}{6}$, and places it in the appropriate box above. The entire group should then examine the two bars. Are the fractions equivalent? Compare the shaded areas in each bar. Are the areas the same? Pick another bar that has the same shaded area as $\frac{2}{3}$ and $\frac{4}{6}$ and place it in the last box. Write the name for the fraction to the right of the box. Are the fractions equivalent?









2. Next, one group member picks the bar that represents $\frac{1}{2}$ and place it in the appropriate box below.

$\frac{1}{2}$

Taking turns, choose three other bars that are equivalent to $\frac{1}{2}$ and place them in the boxes above. Write their fraction names to the right of the appropriate box.

Study the fraction names for these equivalent fractions. How are the numerators and denominators related? Use complete sentences to write your answer.

Write fractions equivalent to $\frac{1}{2}$, but with denominators specified. Check your answers with the other group members, and discuss any answers that do not match. If necessary, consult with another group until an agreement is reached on the correct answers.

$$\frac{\quad}{10}$$

$$\frac{\quad}{18}$$

$$\frac{\quad}{24}$$

$$\frac{\quad}{100}$$

3. Next, consider the fraction $\frac{3}{9}$. In turn, each group member should look through the pile of fraction bars, and find the bar that represents $\frac{3}{9}$, and two other bars that have the same shaded area as $\frac{3}{9}$. Place them in the boxes below, and write the fraction names. Which fraction is simplest? Why?

$$\frac{3}{9}$$

4. Finally, one group member should draw the bar that represents $\frac{9}{12}$.

Another group member then simplifies $\frac{9}{12}$ using the process of multiplying by one, as shown in Section 2.1 of your textbook. Write the simplified fraction below, and the third group member should find the fraction bar that represents it.

Compare this fraction bar with the one drawn for $\frac{9}{12}$. Are the shaded areas the same? Why or why not? Be sure to use complete sentences for your answer.

Conclusion	Equivalent fractions are represented by the same shaded area on a fraction bar. Therefore, to find equivalent fractions, you multiply by a form of 1, as described in Section 2.1 of your textbook. Conversely, to write a fraction in simplest form, you would reverse the process, and remove a factor of 1.
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Activity 2.3 Arrange sockets and drill bits in fractional sizes from smallest to largest.

Focus	Order of fractions
Time	15–20 minutes
Group size	3
Materials	Socket set (optional), drill bit set (optional)
Background	In the United States, tools are typically classified using either English or metric units. For example, socket sets are sold in fractional sizes (fractions of an inch), or metric sizes (millimeters). When fractional sizes are used, it is important to be able to compare the relative size of each tool by comparing the fractions.
Instructor notes	Copy the page of socket and drill bit sets on card stock, and cut out them out. Each group will need one set of each type. You can also ask the students to bring socket sets and drill bits to class.

1. Each group member takes one of the sets and arranges the tools in order from smallest to largest. Write down your result below.

Set _____

smallest

largest

2. Now, pass your paper to the group member to your right and check the result from step 2 on the paper you receive. If you disagree with the result, discuss this with the paper's owner and try to resolve any discrepancies.
3. Next, we will work with the drill bit sets (Sets B and C) only. Mix up the drill bits in the two sets; your group should have a total of 30 drill bits. Deal 10 bits for each group member. One group member starts by choosing one of his or her bits, and placing it on the table. The next group member to the right chooses one of his or her drill bits, and places it next to the bit already on the table such that the smaller bit is to the left. He or she will need to decide which bit is smaller. Continue in turn, with each group member choosing a drill bit from his or her stack, and placing it in the correct position by the other bits already on the table.

4. When all group members have finished placing their drill bits on the table, write down the result below.

5. Compare your group's result with the results of the other groups. Resolve any discrepancies by discussing them with each other.

6. The final step in this activity is to rewrite each fraction in step 4 in terms of the least common denominator of all the fractions. Multiply each fraction by 1, using the appropriate notation, n/n . Write the result below.

7. Compare the lists in steps 4 and 6. Which list is easier to read? Why? Use complete sentences in your answer.

The list in step 4 is used by manufacturers to classify drill bits. Why do you suppose a manufacturer would choose the list in step 4 over the list in step 6?

Conclusion	Fractions in everyday life are typically given in simplified form. Thus, it is important to be able to compare fractions with different denominators. Remember to multiply by 1 to make the denominators the same. The skills learned from this activity can help you do this.
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SET A: SOCKETS

$\frac{3}{8}$	$\frac{13}{16}$	$\frac{9}{32}$	$\frac{5}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{15}{16}$	$\frac{9}{16}$
$\frac{1}{2}$	$\frac{11}{32}$	$\frac{7}{32}$	$\frac{5}{16}$	$\frac{7}{8}$	$\frac{7}{16}$				

SET B: SMALL DRILL BITS

$\frac{15}{64}$	$\frac{5}{32}$	$\frac{1}{4}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{13}{64}$	$\frac{1}{8}$	$\frac{7}{64}$
$\frac{9}{64}$	$\frac{1}{16}$	$\frac{11}{64}$							

SET C: BIGGER DRILL BITS

$\frac{13}{32}$	$\frac{23}{64}$	$\frac{7}{16}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{19}{64}$	$\frac{5}{16}$	$\frac{17}{64}$	$\frac{3}{8}$	$\frac{29}{64}$
$\frac{1}{2}$	$\frac{25}{64}$	$\frac{11}{32}$	$\frac{21}{64}$	$\frac{27}{64}$	$\frac{15}{32}$	$\frac{31}{64}$			

Name

Section

Date

Activity 2.4 Add and subtract mixed numerals using fraction bars.

Focus	Addition and subtraction of mixed numerals
Time	10–15 minutes
Group size	3
Background	Addition and subtraction of mixed numerals involves the concepts of equivalent fractions, borrowing, and converting from fractional notation to mixed numerals. This activity is designed to help you visualize these concepts by using fraction strips to represent the mixed numerals.
Instructor notes	Copy the next page onto card stock, and cut out the fraction bars. Each group will need three sets of bars. Cut out each fraction bar, and also cut along the black lines inside each bar. You can also purchase fraction bars.

1. One group member will be the banker. This person keeps all the fraction pieces, and hands pieces to the other two group members at the appropriate times. The other two group members will take the roles of the top and bottom players in the addition and subtraction problems. The top player will be responsible for the first (or top) number in the problem, while the bottom player takes care of the second (or bottom) number. Write the names of each group member next to his or her designated role.

Banker _____

Top Player _____

Bottom Player _____